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SENSITIVITY TO VISUAL MOTION IN STATISTICALLY DEFINED DISPLAYS

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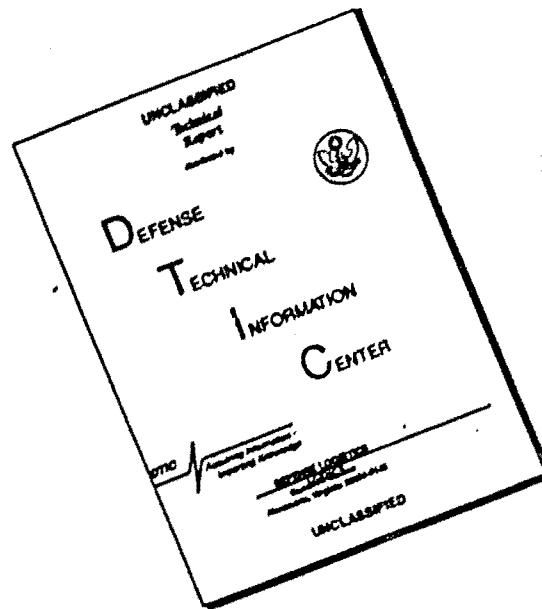
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This project explored several related aspects of how human observers perceive moving targets, particularly targets whose motion is "apparent" rather than continuous. <u>ORIGINS OF GLOBAL MOTION PERCEPTS.</u> When several different, local motion vectors are intermixed the result may be a percept of global, coherent motion. We exploited this discovery in order to develop a better understanding of the mechanisms that support the perception of motion. Our experiments used specially developed stimuli presented by computer control. (continued)		

The stimuli were random-dot cinematograms made up of 512 elements (bright dots on a cathode ray tube). From one frame of the display to the next, each element took an independent, random walk. All steps in the random walk were of constant size and the directions of these steps were drawn from a uniform distribution.

When shown stimuli in which different, local motion vectors were mixed, observers tended to see a global, coherent flow along the mean of the uniform distribution of directions. This perceptual tendency varied inversely with the range of the distribution. Standard psychophysical techniques were used to obtain psychometric functions for cinematograms having various step sizes and spatial densities of their elements. A wide range of conditions produced results that were consistent with a modified version of S. Ullman's "minimal map theory" of motion correspondence.

SIZE FACTORS IN APPARENT MOTION. We tested the idea that the system creating the percept of motion makes use, at an early stage, of information about size. The size information was presumed to arise from size-tuned mechanisms with fairly broad bandwidths in the domain of spatial frequency. To test this hypothesis, we used stimuli whose luminance profile was a difference of Gaussians (DOGs). Such stimuli are spectrally band-limited and therefore should differentially stimulate size-tuned mechanisms.

In one study, adjacent DOGs of varying size were alternated in a simple apparent motion display. When DOGs were of the same size, they were more likely to elicit reports of motion. This preference for size similarity was found under a range of display conditions. The generality of this finding was tested under other conditions. The central DOG stimulus alternated with two flanking DOGs. The central DOG could participate in motion with one, both, or neither of the flanking DOGs. The size relations among the three DOGs were varied and observers' reports of apparent motion recorded. In general, the relations discovered with the two DOG case held for this more complex display. A mathematical model was developed to account for the results. The model conceptualizes apparent motion as resulting from a competition between simultaneous tendencies to see motion in several different directions. The model supports the idea that an early stage in the processing of apparent motion is the extraction of information about the visual size of the stimuli.

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# COHERENT GLOBAL MOTION PERCEPTS FROM STOCHASTIC LOCAL MOTIONS

## Introduction

The combination of several different motion vectors can produce a percept of coherent motion in a single direction. For example, if two sinusoidal gratings of similar spatial frequencies move in different directions, they may appear to cohere into a single moving checker-boardlike pattern (Adelson and Movshon, 1980). Also, if contrast is near threshold, two spatially interspersed random dot patterns moving in orthogonal directions can generate a percept of motion along the mean of the two directions (Levinson, Coyne and Gross, 1980).

Ullman (1979) has demonstrated that many motion percepts, including the result of combining several different motion vectors, can be explained in terms of purely local interactions. The spatial frequency selectivity of coherent unidirectional motion for moving sinusoidal gratings persuaded Adelson and Movshon (1980) that mechanisms which generate the percept of coherent motion operate on the responses of spatial frequency channels. Models of spatial vision that are formulated in terms of spatially localized, spatial frequency channels at each point in visual space have met with considerable success (e.g. Wilson and Bergen, 1979). We were therefore interested in how a coherent global percept could result from the combination of localized motion vectors.

To explore the role of spatially localized processing in the perception of global, coherent motion, we used moving random dot kinematograms. Such kinematograms can be generated according to

diverse rules, resulting in as many different types of stimuli. In one common type, large subsets of the dots move in one direction. But such stimuli would not be appropriate for our purposes: the contribution of the local motion of individual dots to the global percept is obscured by the redundancy of multiple motion vectors in the same direction. Instead, we developed a kinematogram in which the direction of motion of each dot is independently defined. The stimuli were constructed in the following manner. Initially, dots were distributed randomly over our cathode ray display. Each dot then took an independent 2-dimensional random walk. Though all dots travelled the same distance from frame to frame, the direction in which any dot moved was independent of the directions in which the other dots moved. Further, the direction a given dot moved from one frame to the next was independent of the direction of its previous displacements; the possible directions in which all dots moved were chosen from the same uniform probability distribution.

Not surprisingly, if the range of the distribution of directions extended over all 360 degrees, local, random movement of individual dots was evident. But if the range of the distribution was less than 360 degrees, the pattern could appear to flow en masse in the direction of the mean of the distribution, even though the individual perturbations of the dots were still evident.

We parametrized the probability of seeing a global, coherent percept of unidirectional flow from local motion vectors. To do this, we varied the range of the distribution of vectors and measured the probability of seeing unidirectional flow in a direction along the

distribution's mean. We then investigated the properties of local mechanisms of motion by examining how perceived coherence of motion changed with various local parameters. These parameters included spatial factors, the step size in the random walk and the density of dots across the display, as well as a temporal factor, the duration of the movement.

### Methods

The patterns were generated by a PDP 11/34 computer that passed values through a digital to analog converter for display on a Hewlett Packard 1321A X-Y display with a P31 phosphor. The displayed stimulus was confined to a square region with sides measuring 18.5 degrees. A 'wrap around' scheme caused dots to 'disappear' when displaced beyond the boundary of the square and then 'reappear' at the opposite side of the square. The pattern was viewed through a cardboard mask with a circular opening subtending 16 degrees of visual angle. Subjects fixated the center of the screen; viewing was monocular with the other eye occluded by translucent eye patch.

Each dot measured 0.1 degree in diameter. Though frame duration was always 5 msec, the interframe interval required to generate apparent continuous motion varied with step size; Table 1 lists those intervals for each of the step sizes used. Perception of coherent unidirectional flow varied with the stimulus duration (i.e. the number of frames presented). Therefore, except when we measured perception as a function of the number of frames presented, the stimulus duration was maintained at one second. The reason for choosing this value will be



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made clear in a later section, dealing with temporal properties of the stimulus.

The X-Y display provided the only luminance in the room and subjects adapted to these luminance conditions for five minutes before starting an experimental session. Contrast of the patterns was maintained at twice threshold contrast. At the beginning of each session the threshold contrast was reestablished using a von Békésy tracking procedure (Tynan and Sekuler, 1977). Preliminary experiments indicate that a coherent motion percept could be generated over a wide range of contrasts. However since the temporal conditions for producing coherent motion varied with contrast, we decided to confine the formal study to a single contrast.

In preliminary experiments, a 2-alternative forced choice procedure determined the probability of seeing unidirectional flow along the mean of the uniform distribution of directions. These probabilities were measured as a function of the range of the distribution. Steps covered 0.1 degree and the dot density was 1.6 dots per degree. The results were the same for different directions of the mean (e.g. left, right, oblique, etc.). Therefore, with no sacrifice of generalizability we subsequently concentrated only on the case in which the mean direction was upward with respect to the subject. Data were gathered using a simple yes-no paradigm, in which the observer indicated whether or not a coherent unidirectional flow was evident.

Four subjects were tested, three of whom were naive as to the purpose of the study. The fourth subject was one of the authors.

## Results and Discussion

## EXPERIMENT 1: Step Size

For various step sizes, we first measured the probability of seeing coherent flow in the mean direction (upward) as a function of the range of a uniform distribution of directions. Four subjects participated in the study, under conditions already described.

Figure 1 shows the data from subject SDT for 4 different step sizes: 0.1, 0.9, 1.1 and 1.4 degrees. The percentage of trials on which the subject reported coherent unidirectional flow 'upward' is plotted as a function of the range of the distribution. Note that 'upward' is the mean direction of the distribution of directions. Results fall into two categories, depending on whether the step size is larger or smaller than 1.0 degree. If the step size was greater than 1.0 degree, unidirectional flow was reported only when the range of directions was best close to the mean; directions of motion had to be within approximately 45 degrees of the mean for these step sizes. For step sizes smaller than 1.0 degree, a considerably larger range of distribution of directions could generate a percept of coherent flow. In particular, when the total range of 180 degrees was used with small steps coherent motion was reported almost 100% of the time. Similar results were obtained for all four subjects participating in the study. Those for subject AHA are shown in Figure 2. A striking feature of both figures is that a small, two tenths of a degree change in step size, from 0.9 to 1.1, produces a large lateral shift in the psychometric function, while other changes by as much as eight tenths

of a degree, from 0.1 to 0.9, result in little or no shift.

There is a conceptual impediment to a straight forward interpretation of these results. One can not assume that the perceived path a dot travels is the one which was determined by the random walk prescribed for that dot. It may be that for a given dot, its perceived path is a combination of its own random walk with those for intruding neighbors. This perceptual ambiguity is commonly referred to as the 'correspondence problem' (Braddick, 1982; Marr, 1982). If such confusions did occur, spurious directions of movement could be perceived that were inconsistent with the predefined distribution of possible directions. The probability of confusion will depend of such factors as the step size, spacing or density of dots, and the interstimulus interval (Ullman, 1979). If the spacing among dots is increased while other factors remain constant, it seems reasonable to expect that the probability of confusion among paths should be reduced.

#### EXPERIMENT 2: Density of Dots

In the previous experiment the density of dots was constant, 1.6 dots per square degree for all step sizes. We repeated the experiment at three additional densities 0.8, 0.4 and 0.2 dot per square degree, and several step sizes. Four subjects participated in this experiment. Figure 3 shows the results for step sizes of 0.1 and 0.9 degree obtained from subject SDT. For clarity of presentation the data for each step size have been plotted against a separate abscissa.

No significant change in perceptibility occurs when dot density changes by a factor of eight, from 1.6 to 0.2 dots per square degree, for either step size. This constancy is not evident for step sizes larger than 1.0 degree. As shown for subject SDT in Figures 4 and 5, for step sizes of either 1.1 and 1.4 degrees, decreasing the density of dots increases the tendency to perceive unidirectional flow, permitting unidirectional flow to be seen over a wider range of directions. The dashed line in each figure represents the psychometric function for step size 0.1 degree and density 1.6 dots per square degree taken from Figure 1. For a density of 0.2 dot per square degree the data for both step sizes, 1.1 and 1.4 degrees, are almost congruent with this dashed line. Thus for sufficiently small density of dots, perceptibility for step sizes greater than 1.0 degree is nearly equivalent to that for step sizes less than 1.0 degree.

Two important points follow from the results. First, the fact that spacing of dots can alter perception has important implications for the spatial properties of any hypothesized local mechanisms of motion detection and the 'correspondence problem'. These implications are described below, in the General Discussion. A second implication is more germane to the formulation of the remaining experiments and will be discussed here. For step sizes less than 1.0 degree, the constancy of results over a large range of dot densities suggests that spurious directions of displacement do not significantly contribute to the percept. Thus for small step sizes, the perceived random walks more faithfully reflect the prescribed distribution of directions. Because we wish to draw conclusions based on the assumed perceived distribution of directions, the remaining experiments were conducted under conditions

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for which the perceived distribution of directions would be most consistent with the distribution of directions which define the random walks.

### EXPERIMENT 3: Stimulus Duration

Detectability of unidirectional flow was measured as a function of stimulus duration (i.e. the number of frames presented). For two subjects, the effect of stimulus duration was determined for a step size of 0.9 degree with a dot density of 1.6 dots per square degree. Stimulus durations (number of frames presented) used were 2 frames, and all odd numbers of frames ranging from 3 to 13. For a third subject measurements were made for a step size of 0.1 degree at a dot density of 1.6 dots per square degree. The stimulus durations considered in this case were 6, 12 and 25 frames. The relationship proved to be nonlinear: up to eleven frames the probability of seeing unidirectional flow increased with the number of frames presented; presentation of additional frames beyond eleven did not further augment perceptibility. Figure 6 shows the data for two durations, two frames and eleven frames, with step size of 0.9 degree and density 1.6 dots per square degree. It should be noted that the previous experiments discussed and those in the remainder of the paper were conducted using a stimulus duration for which perceptibility is in the asymptotic region.

In analyzing temporal summation for our display, it is important to note that its local motion vectors are distributed in the visual field and this distribution varies with time. We therefore wondered whether the perception of coherent motion depended only on the set of

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directions present from frame to frame or if it also depended on the particular path each dot took over time. For example, would consecutive steps by the same dot in the same direction over a number of frames be more significant to perception than if these individual steps were spatially separated over successive frames?

#### EXPERIMENT 1: Temporal Summation

To examine if spatial factors contribute to temporal summation, we compared perceptibility of coherent motion under two conditions. The first condition used stimulus patterns consisting of two sets of spatially interspersed random dots. For one set of dots (denoted as 'noise') the distribution of directions was uniform over all possible 360 degrees of directions; for the other set (denoted as 'signal'), dots moved only in a single direction; upward on the display. The set assignments of the dots remained the same over all frames presented, so that some dots moved upward frame after frame while other dots moved randomly frame after frame. We'll call this condition the 'Separate' case.

The second condition was identical to the first except in one aspect: in each frame, the particular dots constituting the signal set and those constituting the noise set were chosen independently of the dot assignments to the two sets in previous frames. Though the proportion of dots constituting the signal remained constant over all frames in this condition, there were not two disjoint sets of dots, one signal and one noise, as in the first condition. We'll refer to this condition as the 'Combined' case.

In both conditions, for a given proportion of dots which made up the signal, the distribution of possible directions from one frame to the next is the same. For the Separate case, the probability that any dot made  $N$  consecutive steps in the 'upward' direction is equal to the proportion of dots which are signal; for the Combined case the probability that any dot makes  $N$  consecutive steps in the upward direction is the proportion of dots in the signal raised to the power  $N$ .

The probability of seeing unidirectional coherent flow upward for both conditions was measured as a function of the proportion of the total number of dots which were in the signal. The step size was 0.9 degree and density was 1.6 dots per square degree. Two subjects participated in the study.

As shown in Figure 7, there is no significant difference in the perception of coherent unidirection flow upward between the separate and combined cases. The results indicate that temporal summation over frames is critically dependent only on the distribution of directions of motion present from frame to frame. We can conclude that temporal summation does not depend on the spatial relationship between local motion vectors over time.

#### General Discussion

As we noted before, it is not possible to know a priori whether the perceived path a dot travels is identical to the random walk prescribed for that dot. Consider two successive frames,  $N$  and  $N + 1$ . For a given dot,  $A$ , on frame  $N$  we can define its 'correspondent dot',  $B$ , on frame  $N$

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+ 1. E is the dot on the frame  $N + 1$  to which A is perceived to move between frame N and  $N + 1$ . If A's correspondent dot is the one that was determined by the random walk prescribed for A, the correspondent dot is said to constitute a 'match'; if A's correspondent dot is not the one defined by A's random walk, the correspondent dot is said to constitute a 'mismatch'. Such mismatches produce spurious directions of motion that could be inconsistent with the predefined distribution of possible directions.

Mismatches are the result of the perceptual confusion of random walks prescribed for different dots. Decreasing the spatial density of dots should reduce the possibility of such confusions. If spurious directions of motion due to mismatches do contribute to a percept of coherent, unidirectional motion then a change in the density of dots alone should alter the psychometric function. Our experiments showed such changes. For step sizes greater than 1.0 degrees, decreases in dot density, increases the probability of perceiving coherent flow (see Figures 4 and 5). In these conditions unidirectional flow was perceived over a wider range of distribution of directions at the lower dot densities. This suggests that spurious directions of motion due to mismatches may contribute to the percept of coherent motion.

However our experiments also contained conditions in which the psychometric function was not affected by changes in dot density. For step sizes less than 1.0 degree, a change in dot density by a factor of eight, from 0.2 to 1.6 dots per square degree, did not alter the detectability of coherence (see Figure 3). This suggests that for the smaller step sizes, only the directions of local motion determined by



the predefined distribution of directions significantly contribute to the perception of the unidirectional, coherent motion. Mismatches appear to be minimized or nonexistent for these small step sizes. It should be noted that at a density of 0.2 dots per square degree, perception of coherent motion for steps greater than 1.0 degree is equivalent to that for the steps less than 1.0 degree (see Figures 4 and 5).

Since mismatches are minimized for the smaller step sizes at all dot densities and at the lowest dot density for the larger step sizes, it seems reasonable to speculate that the correspondence between dots on successive frames is based on a nearest neighbor relationship. In this view, the correspondent dot will be the dot on the next frame that is closest. If the correspondent dot constitutes a match then by definition the perceived distance moved is the step size. Table 2 lists the probability that the distance from a given dot on a frame to the nearest dot on the next frame is less than the step size. If a dot is always perceived to move to the nearest dot on the next frame, the Table gives the probabilities that the correspondent dot will not be the one prescribed by the random walk. This is the probability of a mismatch occurring. For each of the step sizes 0.1, 0.9, 1.1 and 1.4 degrees, this probability is shown for two different dot densities: 1.6 and 0.2 dots per square degree.

We tried to determine whether the probability of mismatch could explain the variation in the probability with which coherent flow was seen. For a step of 0.1 degree the probability of a mismatch is extremely small, less than 0.05 at dot densities of both 1.6 and 0.2

dots per square degree (see Table 2). This is consistent with the results shown in Figure 3 for this step size, a decrease in dot density from 1.6 to 0.2 dots per square degree did not alter the psychometric function. With step sizes of 1.1 and 1.4 degrees, the same decrease in dot density reduces the probability of mismatch from 0.99 to 0.53 and 0.71, respectively (see Table 2). With such a large change in the probability of mismatch one would anticipate a significant alteration in the psychometric function with the same change in dot density. As shown in Figures 4 and 5, for both these step sizes this decrease in dot density produces a large increase in the tendency to see unidirectional coherent motion (Figure 4 and 5). For a step size of 0.9 degree, decreasing the dot density from 1.6 to 0.2 dots per square degree, reduces the probability of mismatch by an even larger amount, from 0.98 to 0.40 (see Table 2). As for step sizes 1.1 and 1.4, with such a substantial change in the probability of mismatch, one would expect to find an alteration in the psychometric function with the same change in dot density. However as shown in Figure 3, with the 0.9 degree step size, the probability of seeing unidirectional coherent motion was unaffected by this change in dot density. We suggested above that a variation in the confusability of various random walks could explain why a change in dot density affected the probability of seeing coherent flow. If this explanation is correct, our results for step sizes of 0.9 degree or larger contradicts the hypothesis that confusability - or its inverse, correspondence - is determined strictly by nearest neighbor relationships. Braddick(1973) and Ullman(1979) arrived at similar conclusions regarding the utility of a nearest neighbor basis for the correspondence process.

To deal with this inadequacy, Ullman (1979) has proposed a 'minimal cost theory of motion correspondence' to account for the perceived direction of motion of each element in multi-element motion stimuli. According to the theory, each element (in our case, each dot) is assigned a 'cost function' that determines the probability that a dot will appear to move at a particular velocity. Since the temporal characteristics of all of the elements in our stimuli are the same, we can replace velocity with distance travelled by a dot to simplify the discussion. According to Ullman, the cost function is identical for each element. The distance each element or dot will be perceived to move from one frame to the next will be the distance that minimizes the 'total cost' over all elements in the stimuli. Preliminary results suggest that the functional form of the cost function will be sigmoid (Ullman, 1979, page 118).

Consider a sigmoid cost function that increases with distance travelled and has the sharply rising portion of the sigmoid between 0.9 and 1.1 degrees. For step sizes of 0.1 and 0.9 degree, the 'over all cost' will be minimized by having the dots move from frame to frame the distance prescribed by the predefined random walk. The path each dot is perceived to travel would then be the one defined by the prescribed random walk, as such the number of mismatches would be minimized for all the dot densities considered. For step sizes of 1.1 and 1.4 degrees, it will be more cost efficient to have the dots move distances less than 0.9 degree from frame to frame where ever possible. At the higher dot densities this would result in a significant number of mismatches. As dot density is decreased, the possibility of having a

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dot closer than 0.9 degree as the correspondent dot would be reduced, thereby reducing the number of mismatches. At the lowest dot density, each dot would be perceived to travel according to its predefined random walk. It can be seen that by the appropriate choice of cost function, the results of the first two experiments would be consistent with the minimal map theory of motion correspondence proposed by Ullman (1979). The parameters of the cost function will provide constraints for spatially localized mechanisms of motion perception.

Irrespective of the mechanism of correspondence between the dots on successive frames, the correspondence process alone is not sufficient to explain the generation of a unidirectional coherent percept of motion from local motion vectors. Our data suggest that for step sizes less than 1.0 degree and dot densities of 1.6 dots per square degree or less, only the directions of local motion determined by the predefined distribution of directions significantly contribute to the perception of coherent flow. We also found that although temporal summation occurred in a nonlinear manner over frames, it depended only on the set of directions of motion present from frame to frame. Taken together, these two results are consistent with the idea that directions of the individual steps are independently detected and that these responses are then pooled over time and space to generate the perception of coherent motion.

From the results of Experiment 1, we know that for a step size less than 1.0 degree and dot density 1.6 dots per square degree, a uniform distribution of directions with range 130 degrees generates a percept of unidirectional, coherent motion along the mean for nearly 100% of

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the trials (see Figures 1 and 2). Consider, as usual, the mean of the distribution to be upward with respect to the subject. For this stimulus, on each successive frame, each dot will be above or at least level to its position on the previous frame. (The majority of the dots will of course be translated horizontally on successive frames as well.) If the direction of the individual steps are independently detected and then the responses pooled, then the simple failure to perceive a dot below its previous position may be sufficient to generate the percept of coherent, unidirectional flow in the upward direction. We tested the idea. For the distribution of directions with a range of 180 degrees, the probability of seeing unidirectional flow along its mean was measured as a function of the range of a uniform distribution of directions that was deleted from the center of the original distribution. For each of the distributions constructed in this manner, every dot will be above or at least level with its position on the previous frame. The step size used was 0.9 degree and the dot density was 1.6 dots per square degree. Data were obtained for two subjects and the results are shown in Figure 8.

The percentage of trials on which the subject sees coherent, unidirectional upward flow is plotted as a function of the range of the distribution of directions deleted. As shown in Figure 8, if the directions of motion within 20 degrees of the mean were removed from the initial distribution, the frequency of seeing coherent flow along the mean is reduced to 50%. It should be noted that for this particular distribution, more than 99% of the dots will be above their position on the previous frame, while less than 2% will be level with its previous position. It is clear that the presence of local motion vectors all of

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which have a component in the direction mean is not sufficient to ensure a percept of coherent unidirectional flow. To generate the percept, directions of local motion vectors in the neighborhood of the mean must also be present. This suggests that the percept results from the nonlinear spatial pooling for responses of direction selective mechanisms that are tuned to the mean direction of the distribution.

Many people have used cross correlation as a framework within which to model direction selectivity; in this scheme spatial samples displaced by distance  $dx$  with a time lag  $dt$  are cross-correlated (Reichardt and Varju, 1959; Poggio and Reichardt, 1973). For the stimulus patterns we have considered, with directions of motion chosen from a uniform distribution, the cross correlation between two successive frames is the same in all directions present in the distribution. Thus, any correlation mechanism that could account for the experimental results must be selectively sensitive to a range of directions. In the visual spatial domain this implies that the cross correlation is applied to the output of spatially localized mechanisms each of which has a receptive field that is orientation selective.

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TABLE 1

DURATION OF INTERFRAME INTERVAL REQUIRED TO GENERATE  
APPARENT CONTINUOUS MOTION FOR A GIVEN STEP SIZE

STEP SIZE (degrees)	INTERFRAME INTERVAL (msec.)
0.1	35.
0.3	50.
0.5	70.
0.8 or greater	90.

TABLE 2

THE PROBABILITY THAT THE DISTANCE FROM A GIVEN  
DOT IN A FRAME TO THE NEAREST DOT IN THE  
NEXT FRAME IS LESS THAN THE STEP SIZE

STEP SIZE (degrees)	DENSITY OF DOTS (dots per square degree)	
	1.6	0.2
0.1	0.05	0.01
0.9	0.98	0.40
1.1	0.99	0.53
1.4	0.99	0.71

The distribution of dots on each frame is Poisson with parameter,  $d$ , the density of dots per square degree. The probability that the distance from a given dot on a frame to the nearest neighbor on the next frame is less than the step size,  $s$ , is given by

$$1 - \text{EXP}(-\pi * d * s * s)$$

## FIGURE CAPTIONS

FIGURE\_1: The percentage reports of unidirectional, coherent flow in the upward direction as a function of the range of a uniform distribution of directions. The mean of the distribution was in the upward direction; the range is given in degrees. Data were obtained for 4 different step sizes 0.1, 0.9, 1.1 and 1.4 degrees. The dot density in each case was 1.6 dots per square degree. The results fall into 2 categories, depending on whether or not the step size is larger or smaller than 1.0 degree. (Data for subject SDT).

FIGURE\_2: Same as Figure 1, except data for subject AHA.

FIGURE\_3: The percentage reports of unidirectional, coherent flow in the upward direction as a function of the range of a uniform distribution of directions. The mean of the distribution was in the upward direction; the range is given in degrees. Data were obtained for 2 different step sizes 0.1 and 0.9 degree. For both step sizes the measurements were obtained at two dot densities 0.2 and 1.6 dots per square degree. For step size 0.9 degree, measurements were also obtained at dot density 0.3 dots per square degree. The psychometric function for each step size remains essentially unchanged with a change in dot density. (Data for subject SDT).

FIGURE\_4: The percentage reports of unidirectional, coherent flow in the upward direction as a function of the range of a uniform distribution of directions. The mean of the distribution was in the

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upward direction; the range is given in degrees. Data were obtained for step size 1.1 degrees at three different dot densities 0.2, 0.8 and 1.6 dots per square degree. For this step size, perceptibility does change with dot density. With a decrease in dot density, unidirectional coherent flow was perceived over a wider range of distribution of directions. For a density of 0.2 dots per degree the data for a step size of 1.1 degrees is almost congruent with those for step size 0.1 degree and density 1.6 dots per square degree (taken from Figure 1) represented by the dashed line in the figure. (Data for subject SDT)

FIGURE 5: The percentage reports of unidirectional, coherent flow in the upward direction as a function of the range of a uniform distribution of directions. The mean of the distribution was in the upward direction; the range is given in degrees. Data were obtained for step size 1.4 degrees at three different dot densities 0.2, 0.4 and 1.6 dots per square degree. For this step size, perceptibility does change with dot density. With a decrease in dot density, unidirectional coherent flow was perceived over a wider range of distribution of directions. For a density of 0.2 dots per degree the data for a step size of 1.4 degrees is almost congruent with those for step size 0.1 degree and density 1.6 dots per square degree (taken from Figure 1) represented by the dashed line in the figure. (Data for subject SDT).

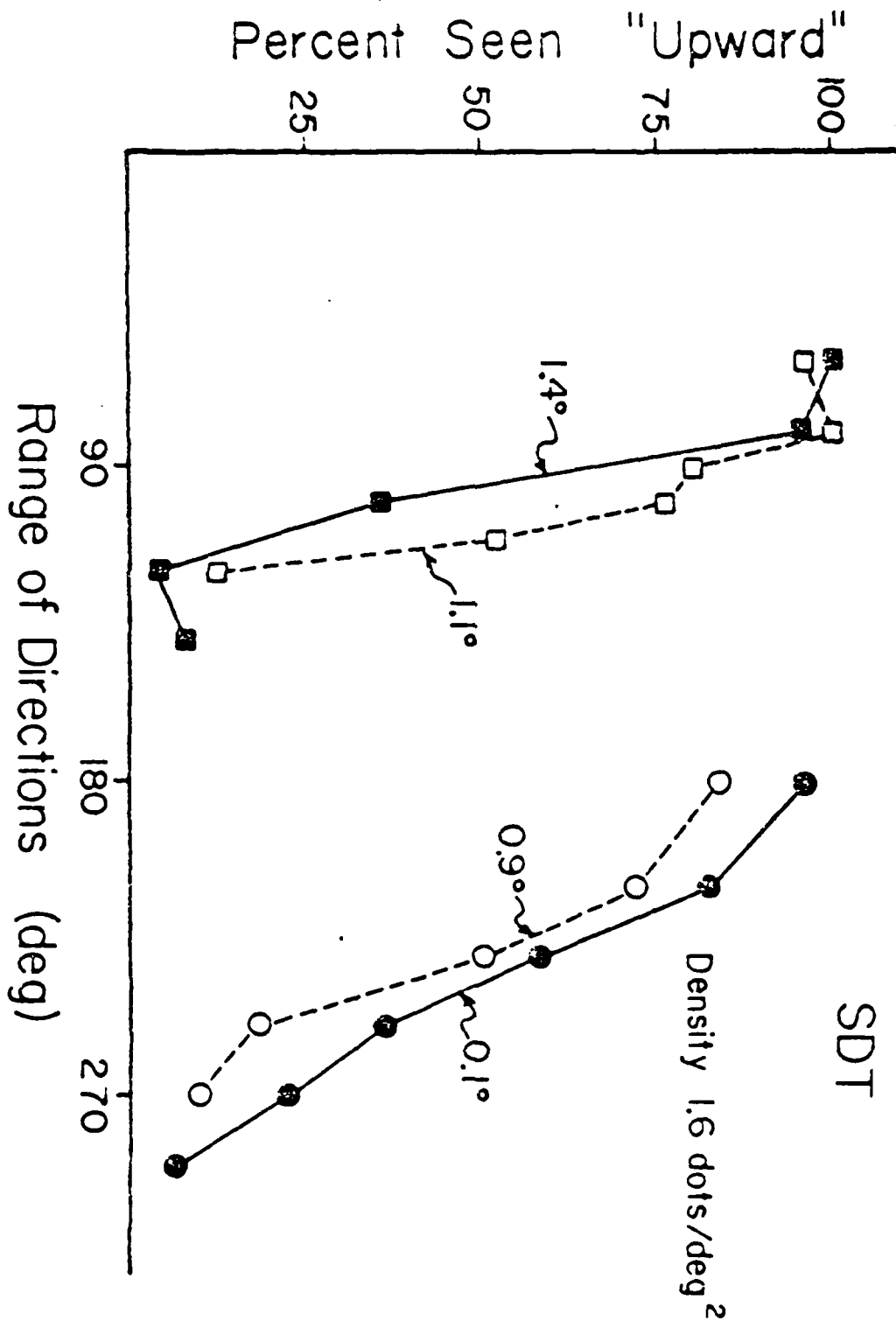
FIGURE 6: The percentage reports of unidirectional, coherent flow in the upward direction as a function of the range of a uniform distribution of directions. The mean of the distribution was in the upward direction; the range is given in degrees. Data are shown for two

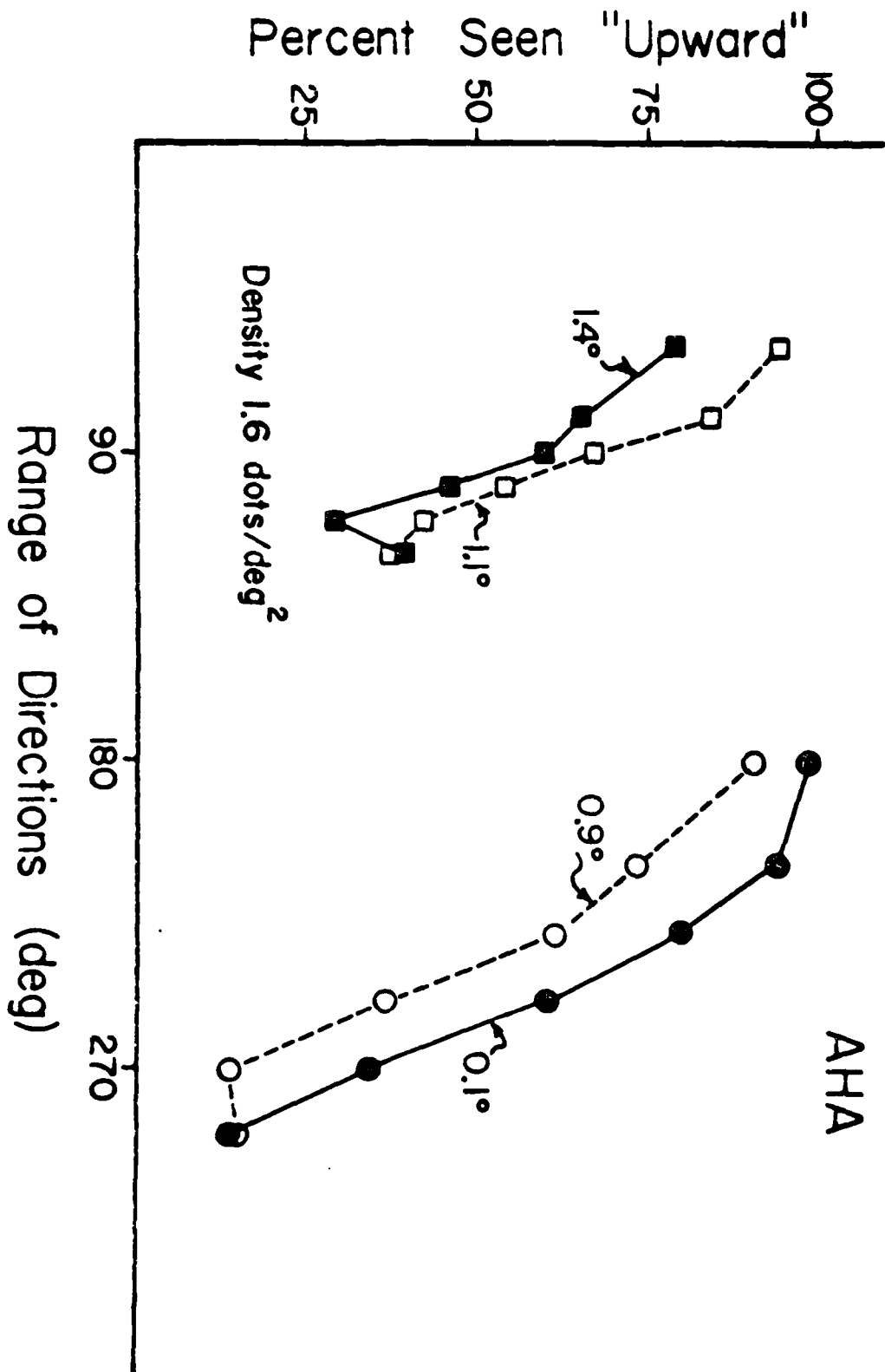
Williams and Sekuler

different stimulus durations: 2 frames and 11 frames. The step size was 0.9 degree and the dot density was 1.6 dots per square degree. Perceptibility increases with the number of frames presented. (Data for subject SDT).

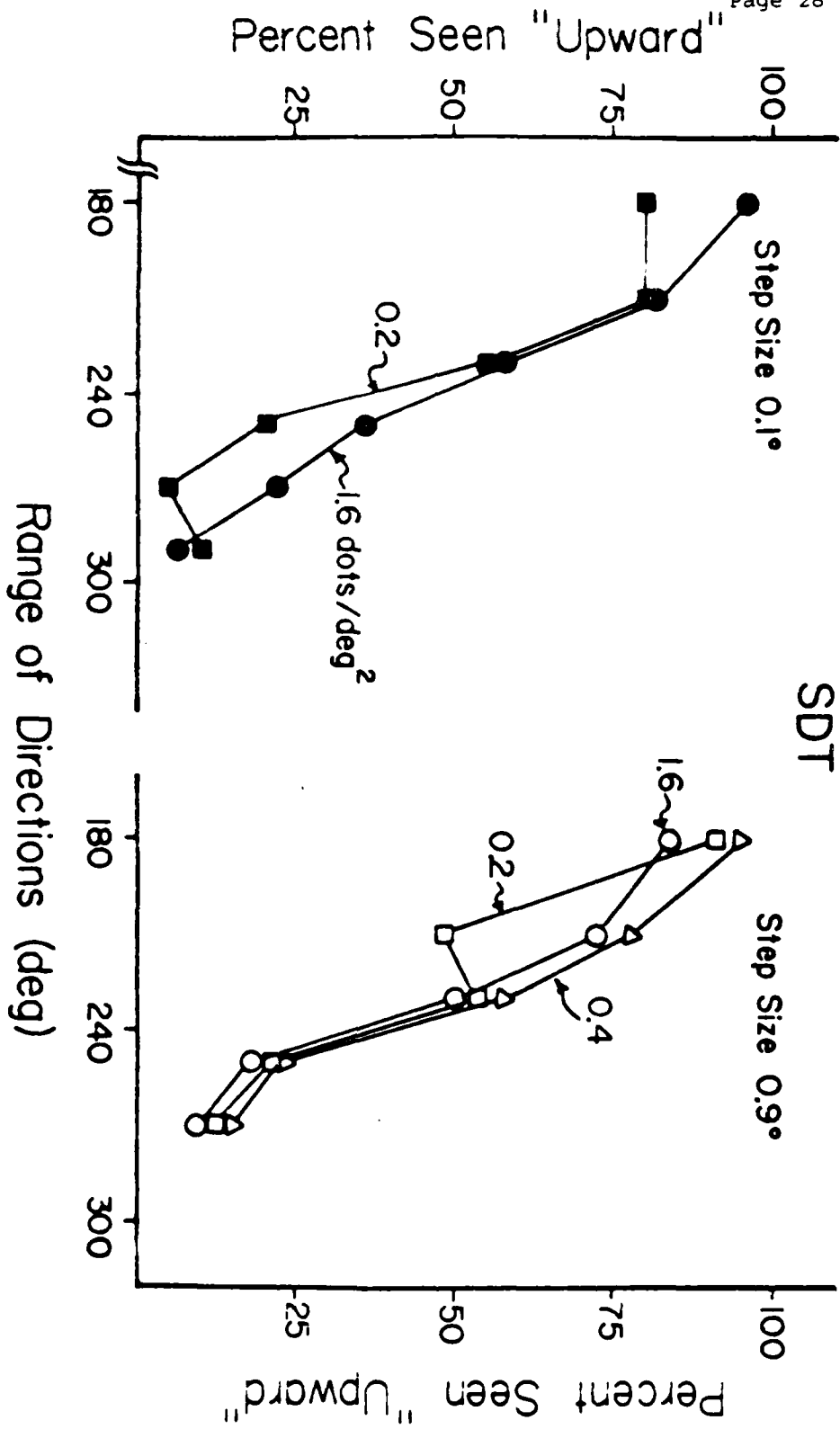
FIGURE 7: The percentage reports of unidirection coherent flow in the upward direction as a function of the percentage of dots in the 'signal set'. Dots in the 'signal set' moved only in the upward direction; dots in the 'noise set' took their directions from a uniform distribution covering 360 degrees. The curve labelled 'Separate Distribution' denotes a condition in which dot allocation to the 'signal set' and 'noise set' did not change for all frames presented. The curve labelled 'Combined Distribution' denotes a condition in which dots are allocated to each set on each frame independently of allocations on previous frames. There is essentially no difference in perception between the two conditions. (Data for subject SDT).

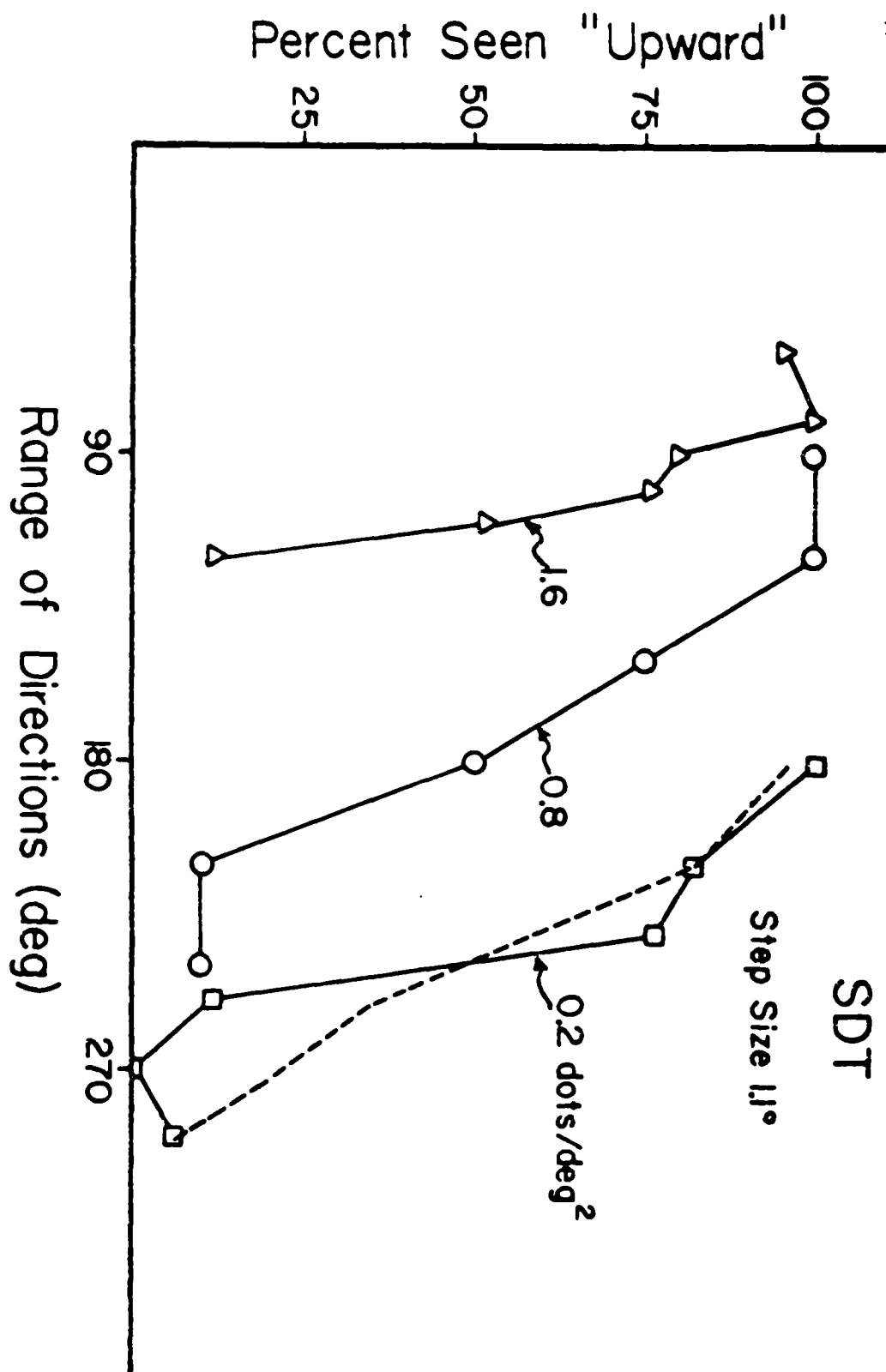
FIGURE 8: The percentage of reports of unidirection coherent flow in the upward direction as a function of the range in degrees of a uniform distribution of directions deleted from the center of a uniform distribution. The distribution, before deletion, covered 180 degrees. (Data for subject SDT).

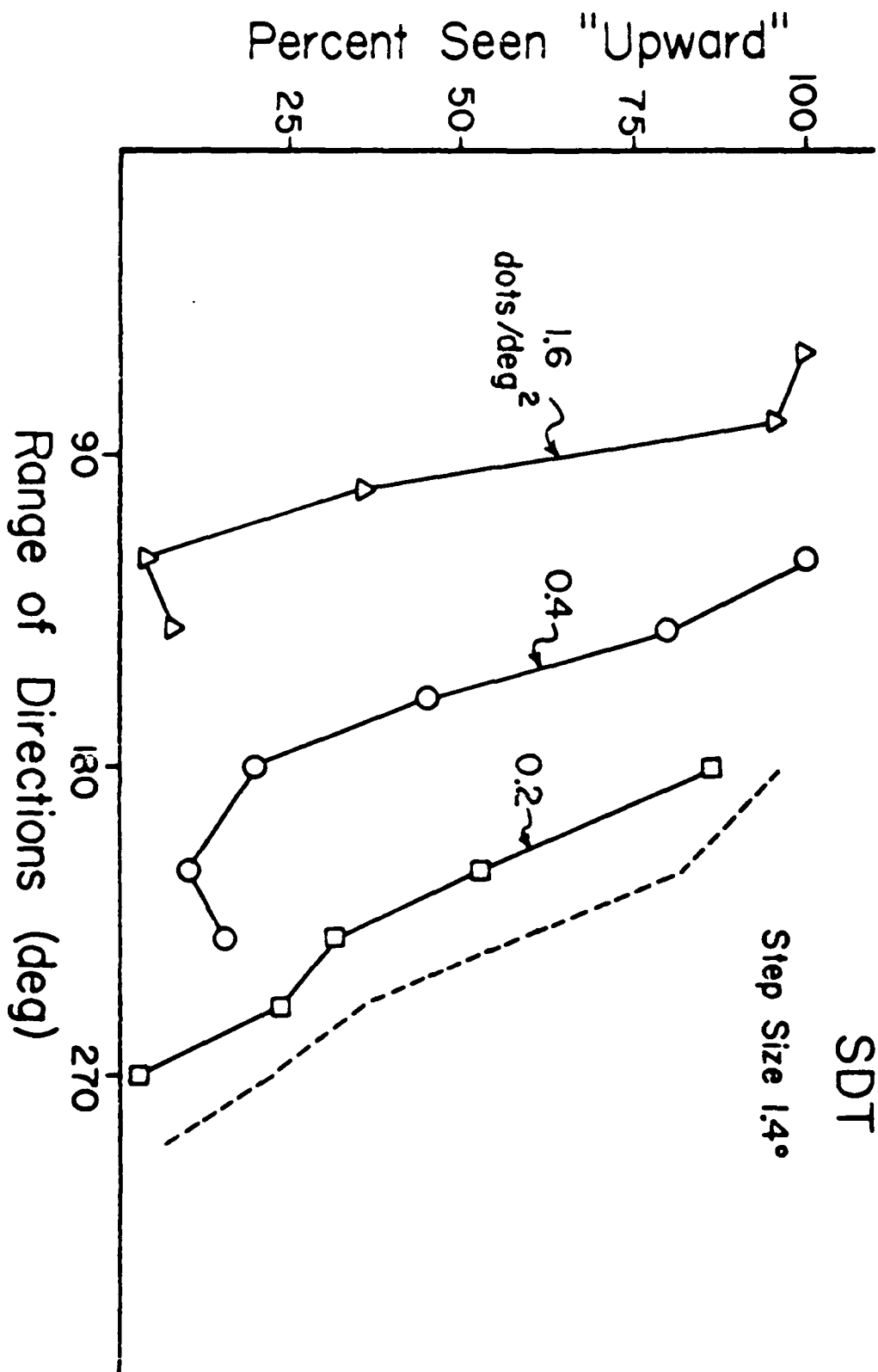


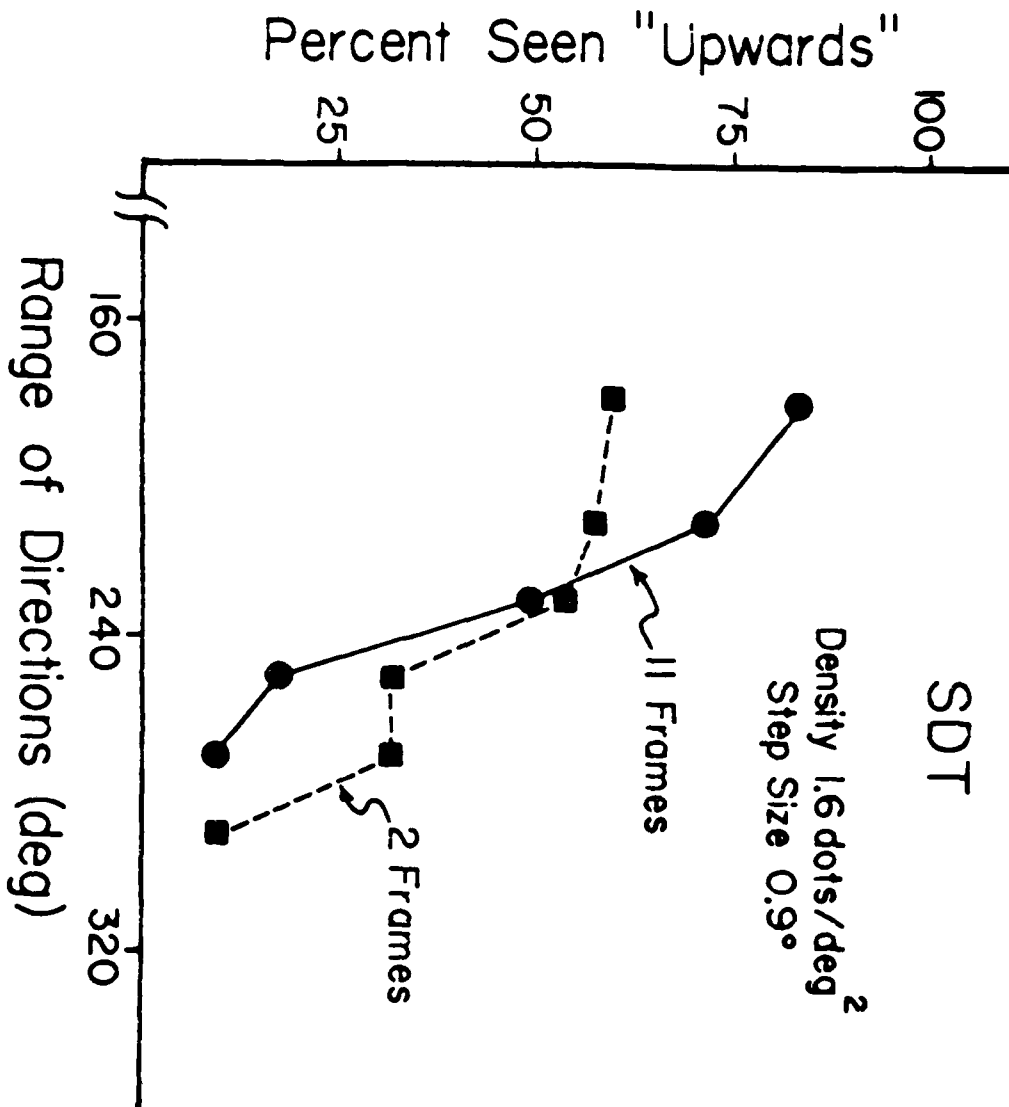


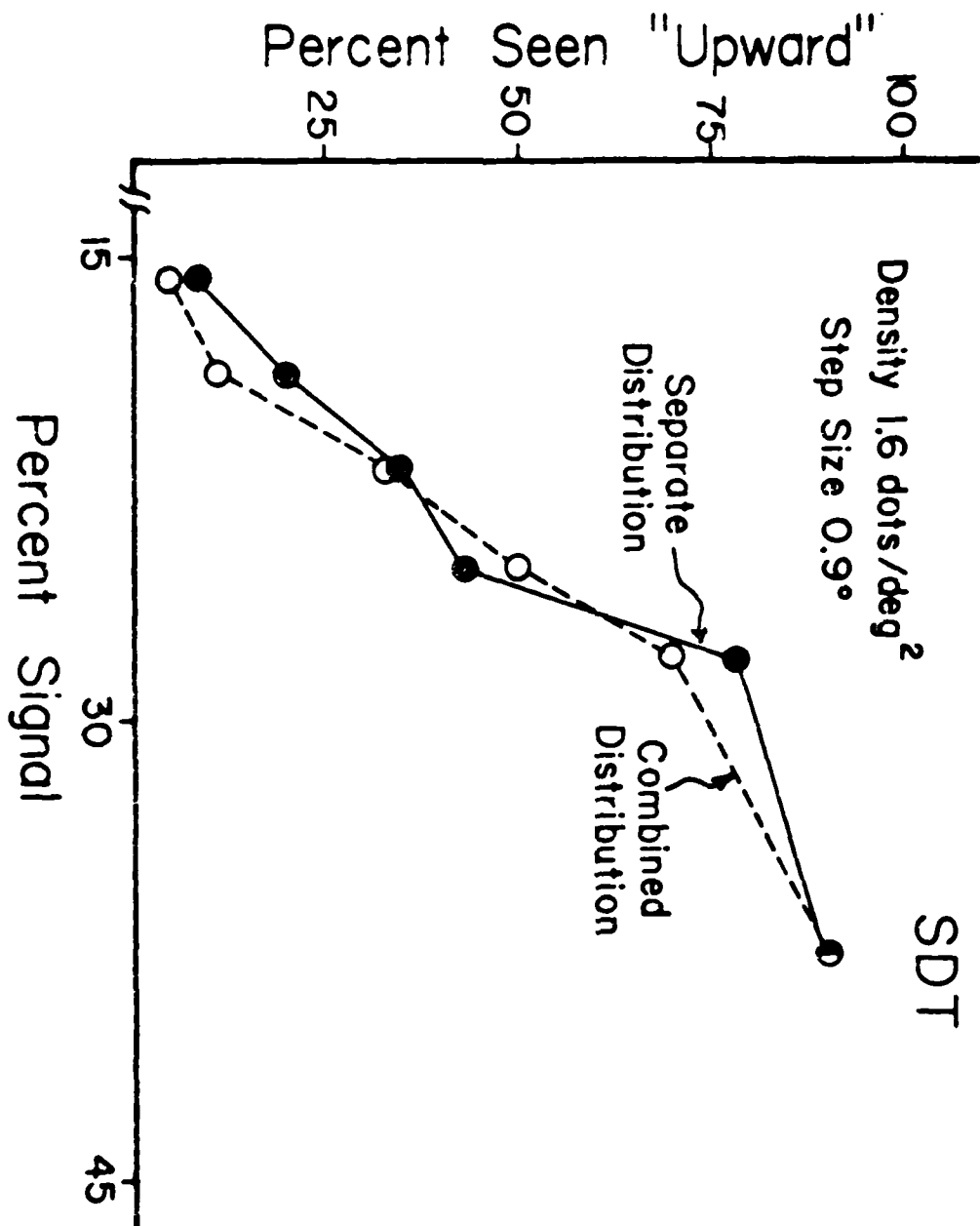


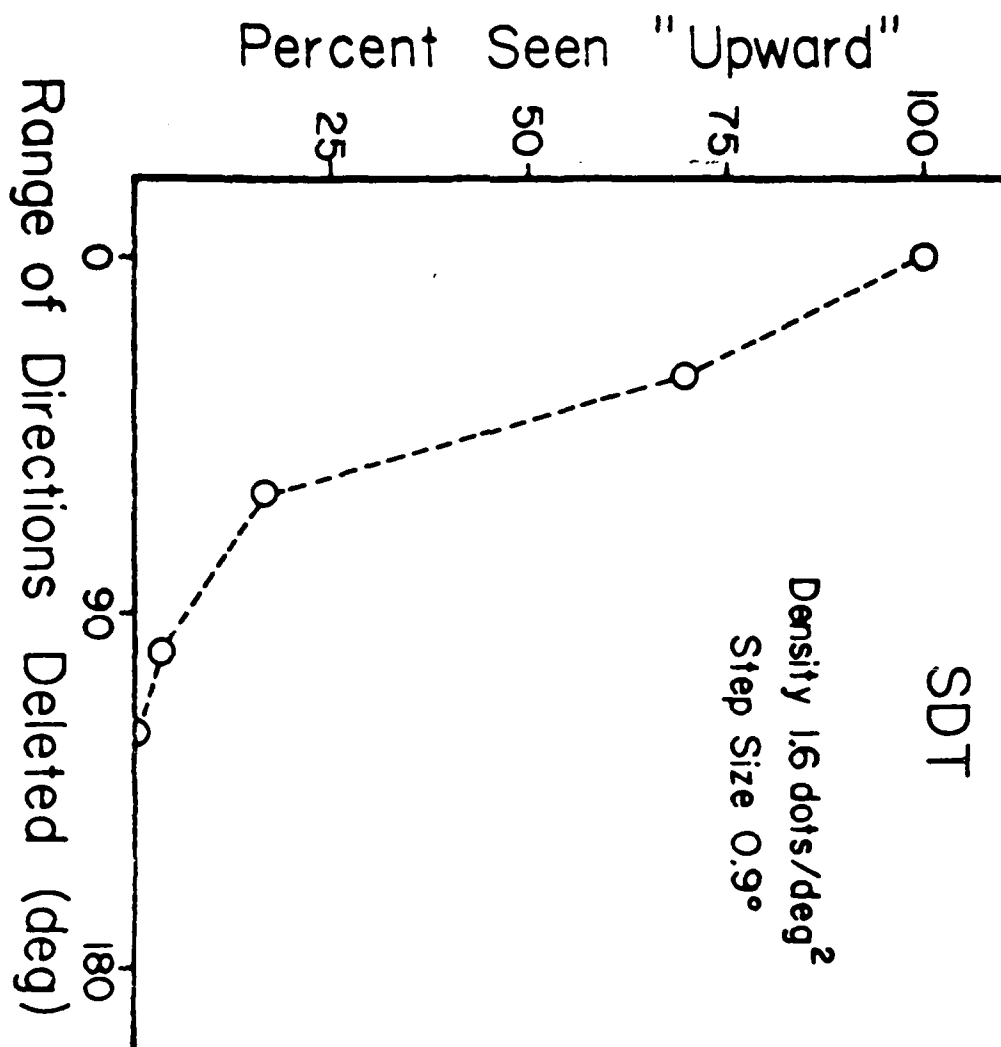












## Size Factors in Apparent Motion

## General Introduction

The nature of the sensation of motion has been debated since at least the time of Zeno whose mathematical paradoxes have taken centuries to tame. Zeno's assertions on motion perception are still interesting. Specifically, he suggested that objects are detected in different places at different times; memory, bridging the gap of time, connects the objects of past and present by inferring motion to resolve their spatial discrepancy. Some light was shed on the subject by demonstrations in the late 19th century (Exner, 1875); adjacent electrical sparks could give rise to a sensation of motion even though they occurred so close in time that their order could not be reliably reported. Clearly, memory could not have any role in the sensation of motion here, or could it? The argument assumes a view of how the mind operates that is different from that prevalent today. Now, it is certain that when past events influence present ones in an orderly fashion, then something analogous to a memory is operating, even if it is not available for introspective interrogation.

The phenomenon described by Exner, in which two stimuli presented in succession give rise to a sensation of motion from the first to the second, has become known as

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apparent motion (AM). This is to be distinguished from "real" motion in which the stimulus follows a continuous path. Apparent motion is amenable to a remarkable variety of experimental manipulations, many of which have been done by now. One of the largest and earliest contributions to this literature is the work of Wertheimer. Considered a seminal paper in Gestalt psychology, Wertheimer's (1912) article opened up the questions of limits on spatial separation, timing, duration and intensity necessary for the production of this illusory motion. Gestalt psychology realized the value of such illusions--that they are not so much defects as logical consequences of the underlying rules employed by the perceptual systems, and as such offer the opportunity to discover those rules.

In regard to apparent motion, one of the key issues of interest to these psychologists was that of phenomenal identity. Without the continuity provided by real motion, the objects in the two frames of an apparent motion display are somehow matched, or identified as a single object in motion. This identification task has come to be known as the "correspondence problem." In displays consisting of single flashes of light, this would hardly seem to be a problem; but the Gestalt psychologists were adept at designing more ambiguous displays, which were nonetheless readily perceived as motion.



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An example where the correspondence problem is really more of a problem is the display of Ternus (1926). Fig 1 shows the two frames of this display, the first on the top and the second on the bottom. When presented in alternation at the appropriate rate, the two dots appear to move as a group, although the central dot actually never changes position. According to the Gestalt thinkers, the fact that this result would not have been predicted by observing either pair of dots in isolation is indicative of the "globality" of the correspondence process. Observations such as this evolved into the principles of Pragnanz (Koffka, 1935), stating essentially that, within the constraints of the information present, the percept formed will maximize such properties as symmetry, simplicity, regularity and so forth.

Due partly to the advances in physiological psychology and to the growing body of research on "real" motion, speculation on apparent motion has grown increasingly mechanistic. Divers researchers have realized the power of relatively simple, physiologically realizable local processes to detect and analyse motion. The basic scheme is exemplified by Barlow and Levick's (1965) model for the directionally selective cells in the rabbit retina (Fig 2). Two spatially separated detectors (R1 & R2) are logically conjoined via an AND gate; one is connected directly (R1) and

the other is connected via a time delay, with its sense inverted (not R2). Objects passing from R2 to R1 will send, simultaneously, signals "true" (from R1) and "false" (the delayed inverted R2) to the AND gate, which will not respond. Stimuli moving the opposite direction do not have this inhibition problem, so the output of the AND gate is effectively directionally selective.

Such a scheme has the obvious advantage of explaining the sufficiency of discontinuous stimuli to produce a sensation of motion. It is also amenable to modifications such as replacing the time delay by an element with low-pass temporal properties, or adding characteristics to explain the course of adaptation or aftereffects. Yet, this sort of motion detector can only suffice at the most primitive levels of any working motion system; without a rather sophisticated algorithm to interpret an ensemble of these simple units, their outputs in response to complex moving scenes would be rather ambiguous. The heart of the problem is that in order to deduce motion from the response of such a detector, one is forced to make the naive assumption that the two inputs were stimulated by the very same physical object on a continuous path; this is not always a reliable assumption. To make the response of a simple motion detector a reliable indicator of motion requires additional information from other motion detectors, memory, etc. In other words, the simple units

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must somehow be wired to solve the correspondence problem.

A number of explicit theories have been suggested for accomplishing this, notably the vector model of Brown and Voth (1937) and more recently an iterative model by Caelli (1980); all are distinguished by working well, but only for a modest range of displays. Despite the failure to produce quantitatively useful models, some progress has been made in understanding the problem at a higher level of abstraction. One line of progress has been a computational theory outlining the goals and resources of the system and some of the logic needed to connect them, but without reference to particular algorithms. An example of such an approach in motion perception is the work of Ullman (1979), and similar work on other visual problems can be found in Marr (1982).

After deciding that motion perception can be conceived as a correspondence problem, the preeminent remaining issue is what is being put in correspondence. It is on this issue that I will present some new empirical evidence. One suggestion for the domain of the correspondence process is raw, gray-level data from the images (Anstis, 1970). Two frames of an apparent motion display could be compared point by point (or perhaps by small "windows" of points) using cross-correlation or differencing techniques, and the match yielding the minimal error would be the one perceived. As

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Ullman points out, a simple gray-level match could not account for what is perceived when figures such as those in Fig 3. are shown. Fig. 3a shows the intensity profile for the first and second frames (top and bottom); each contains a smooth gradient and a fairly sharp edge (marked by a \* here). The smooth gradient in the first frame is in registration with the sharp edge on the second, and vice-versa. A correspondence based on gray-levels alone will show a maximum correlation centered on zero, i.e. no motion should occur when these two frames are alternated. Human observers, however, report motion between the two sharp edges.

The conclusion drawn from experiments such as the one above is that some higher level organization of the raw intensity data takes place prior to the correspondence process. A good deal of attention has been paid to the figural aspects of the stimuli in apparent motion, mostly with null result. Orlansky (1940) made a more or less systematic investigation of the correspondence of various geometrical figures shown in alternation. Squares, circles, triangles were matched with one another. Perhaps surprisingly, the disparate pairings produced good apparent motion; a modest amount of selectivity was demonstrated, however, by measuring the range of interstimulus intervals that would support apparent motion for each pair. Kolers and Pomerantz (1971) performed a somewhat cleaner version of

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essentially the same experiment, using the percent seen in smooth motion as a dependent variable, and found again almost no effect due to figural similarity.

An ingenious study by Navon (1972) presented a subject with several possible paths on which to see the object present in the first frame to move to in the second frame; the objects were different Hebrew letters. Again, no reliable differences were found to suggest that the motion system preferred to match like letters. A similar result was reported by Burt and Sperling (1981) who also used a competing paths technique and simple geometric figures. On the other hand, Frisby (1972) has shown that difference in the orientation of line segments can affect the likelihood of seeing apparent motion. Ullman (1980) provided corroborating evidence along with data indicating that the length of vertical line segments also influenced their correspondence. Thus it would seem that the analysis of motion must precede the organization of the image into complex forms, but probably follows certain more primitive levels of analysis.

Marr (1982) has divided visual organization into several hierarchical levels. This hierarchy starts with the raw intensity values present in the image. Changes in the sign of the slope of these intensity arrays are then noted as "zero crossings". Zero crossings refer to places where the

## Size Factors in Apparent Motion

"non-directional" second derivative, or Laplacian

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

is zero. This may seem like an unlikely operation, but it does have nice properties and can indeed be computed by biologically realistic hardware. Consider that if a Gaussian function were first convolved with the image before computing the Laplacian, it would be mathematically equivalent to taking the Laplacian of the Gaussian and then convolving this with the image. The Laplacian of a Gaussian is a Mexican hat-shaped affair, very similar to the difference of Gaussians curve (DOG) shown in Fig. 4a. It also looks remarkably like the weighting functions of the receptive fields of retinal ganglion cells as described by Enroth-Cugell and Robson (1966), with an excitatory center and an inhibitory surround.

By using different sizes of receptive field, zero crossings can be obtained at varying scales or levels of coarseness (equivalent to blurring the image by convolving with different sized Gaussians). The reports of these zero-crossings can then be combined to form, Marr suggests, the signals of the next level of the hierarchy, the raw primal sketch. Items such as bars, edges and blobs with attributes of position, length, width, orientation and contrast are the primitives of the raw primal sketch. It is this level that Marr explicitly suggests forms the domain of

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the correspondence problem in motion.

Ullman, in his book (1979), has elaborated on Marr's primal sketch suggestion, and has provided some evidence supportive of it. One such piece of evidence is a demonstration that supposedly whole forms can be broken into simpler constituents to satisfy the solution of the correspondence problem. The "broken wheel" demonstration is a variation on the well known wagon wheel effect seen in motion pictures wherein upon reaching a certain angular velocity a spoked wheel appears to change direction. Fig. 5 shows such a wagon wheel in which every other spoke is broken by having a piece removed from its center. By presenting slightly rotated versions of the pattern on successive frames, the wheel will appear to rotate. The solid and dotted lines in Fig. 5 show the first and second frames of such a sequence. At a certain rate of succession however, what is observed is that the wheel seems to split up into three concentric wheels. As indicated by the arrows, the outermost and innermost segments will appear to rotate clockwise, while the central (broken) segment will appear to rotate in the opposite direction; this is consistent with the "minimum distance" principle observed in many ambiguous AM displays, but is surprising in that it suggests that, for the purposes of motion analysis, a whole figure such as the

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spoke really consists of a number of components.[1]

### Experiment One: Introduction & Methods

Although it has been suggested that size is an attribute available in the primal sketch and hence a candidate for influencing the correspondence process in motion, evidence for this is scant. As mentioned before, Ullman did look at the effect of length on vertical line segments in horizontal apparent motion, and noted that paths pairing segments of equal length were preferred to paths pairing segments of unequal length. Kolers (1972) mentioned anecdotally that a square figure, measuring 0.3 deg of visual angle/side was just as likely to appear to move to a like-sized circle as to an identical square; but, when all figures were scaled up to 1 degree, the similar figures predominated in motion. Fernberger (1934) used "wide" and "narrow" bars (no dimensions given) in the Ternus configuration as shown in Fig. 6. When the relative positions of the two sized bars remained the same from first to second frame, the bars appeared to move as a group just as Ternus had reported (6a). When, however, the bars reversed

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1. The minimum distance principle dictates that the mapping chosen minimizes the total distance moved, summing over all the individual paths.



## Size Factors in Apparent Motion

### Experiment One: Introduction and Methods

position so that bars of the same size occupied the central position (Fig. 6b), the central bar remained stationary while the flanking bars moved from side to side. This behaviour is consistent with a size selective correspondence process.

If the size information available in the raw primal sketch is based upon the responses of different sized center-surround operators, then the simplest sort of size difference would be represented by the differential stimulation of the various classes of these size tuned mechanisms. Wilson and Bergen (1979) have suggested that only four classes of size tuned mechanisms are necessary to account for human psychophysical performance for patterns concentrated below 16 c/d. Their conclusions are based on threshold data for various patterns that is consistent with a small number of broadly tuned (bandwidths between 1 and 2 octaves) detectors and certain assumptions about their joint probability distribution. While the number of such classes, and even whether there is a finite number, are still highly debatable topics, there is a vast pool of evidence pointing to size tuned mechanisms. Besides the detection data of Wilson and others, studies of adaptation (Williams, Wilson & Cowan, 1982) and visual masking (Mostafavi and Sakrison, 1976) draw similar conclusions.

## Size Factors in Apparent Motion

### Experiment One: Introduction and Methods

The ideal stimulus, then, would be both localized and band-limited in spatial frequency. In this context, to be localized means that the stimulus will occupy a finite area on our display screen--its place will be unambiguous. To be band-limited in spatial frequency space means that if we were to decompose the stimulus into a sum of sine and cosine waves of different frequencies and amplitudes via Fourier analysis, then above a certain frequency the amplitudes would decrease monotonically and likewise, below a certain frequency the amplitudes would also decrease monotonically.

Although many classes of functions could be devised to fulfill these requirements, the obvious choice is a difference of two Gaussians (DOG). This pattern was used by Wilson and Bergen in the formulation of their model. With the parameters as shown in Eqn. 1 the DOG has a 1.8 octave half-amplitude full bandwidth. A typical DOG and its Fourier transform have been plotted in Figs. 4a and 4b.

$$(1) \quad \text{DOG}(x) = 3\exp(-x^2/\sigma^2) - 2\exp(-x^2/2.25\sigma^2)$$

The center frequency of the DOG depends on the dispersion parameter, sigma. This frequency can be shown to be inversely proportional to sigma. Casual reference following to the "frequency" of a DOG will actually refer to DOGs of the form in Eqn. 1 whose Fourier transforms are centered at

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the specified frequency. DOGs centered at lower spatial frequencies will be larger than those centered at higher spatial frequencies.

The most straightforward way to test whether the size information is used in the correspondence process for motion perception is to present pairs of DOGs of different sizes in a situation known to produce apparent motion between two identical objects. If observers report motion more frequently between similar DOGs than between dissimilar DOGs, then we would have some evidence indicating that this sort of size information is used in the analysis of motion. If, on the contrary, observers show little distinction between different sized pairs and same sized pairs, (as Kolars' subjects failed to do for different geometrical figures), then we might conclude that size as defined in this fashion is not of primary importance in developing a correspondence. This is the essential rationale for the first of two experiments to be presented here.

The factor of interest in this and in the subsequent experiment is the spatial dispersion,  $\sigma$ , of the stimulus. It is important to rule out other factors that may also covary with  $\sigma$ . The total area above zero (where zero represents the mean luminance) is equal to the total area below zero for all DOGs defined as in Eqn. 1, so the mean

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luminance will not change with sigma. The issues of contrast and total energy are not as simple. The contrast or modulation level of a sine wave is fairly straightforward to find because its amplitude is symmetric about the mean. Its contrast is commonly expressed as the difference between the maximum and minimum luminance levels ( $L_{max}$  &  $L_{min}$ ) divided by their sum.

$$(2) \quad \text{Contrast} = (L_{max} - L_{min}) / (L_{max} + L_{min})$$

As you can see, contrast ranges over the closed interval  $[0,1]$ . With functions that don't show the symmetry across the x-axis that sine waves do, the definition of contrast will inevitably be somewhat arbitrary. The definition which Wilson and Bergen used is the maximum luminance minus the mean luminance ( $L_{mean}$ ) all divided by the mean luminance,

$$(3) \quad \text{Contrast} = (L_{max} - L_{mean}) / (L_{mean}).$$

This function also ranges over  $[0,1]$  and, you can see that it is equivalent to (2) when applied to sine waves.

The formulae above were derived from mathematical considerations and not psychophysical ones. It is not a priori certain that patterns with the same calculated contrast will appear to have equal contrast to observers. No published reports are available on the suprathreshold

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apparent contrast of DOGs, and how this may vary, if at all, with their center frequency. Georgeson and Sullivan (1975) carried out extensive experiments on the apparent contrast of sine waves of different frequency and bars of different width. The overriding conclusion of their work is that the visual system shows remarkable contrast constancy over the different sizes; subjective contrast matches between sine waves of different frequency or lines of different width were very nearly veridical. This constancy proved to be largely independent of luminance and position on the retina. The robustness of contrast constancy invites the conclusion that different sized DOGs of the same calculated contrast will share the same apparent contrast. To be certain that Georgeson and Sullivan's results would generalize to DOGs under the conditions of brief presentation to be used in the apparent motion display, some exploratory data was collected.

Observers viewed binocularly the one-dimensional vertical DOG patterns on a Tektronix 606A cathode ray tube from a distance of one meter. At this distance, the screen subtended 6 degrees of visual angle. Patterns were generated by streaming luminance data from a digital computer; the visible part of each frame was composed of 1100 lines, each assigned its own luminance value from the computer's memory. The frame rate was 60 per second. Contrast levels could be varied uniformly over the entire screen by an analog

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multiplier, also under control of the computer. A block diagram of the system with all parameters is shown in Appendix 1.

The mean luminance of the screen was kept at 12.3 candelas throughout; contrast could be varied from 0.0 to 0.44 while remaining well within the linear range of the system. Observers were presented with a series of trials, each starting with a single frame (16.7 ms.) presentation of a particular DOG, a 1 second period of a uniform screen at mean luminance, and finally the single frame presentation of another DOG of a different size. The one second delay between presentations was employed to minimize masking effects. The subject was asked to indicate by pressing a lever which DOG had the greater contrast. The first DOG maintained a standard contrast throughout a session, while the contrast of the second DOG varied from trial to trial with subject responses according to a staircase procedure (Wetherill & Levitt, 1965). The staircase used was designed to track the point at which the subject was just as likely to pick one DOG as the other (50%). Initial changes in the contrast of the second DOG were made in 1.5 dB steps. After a certain number of trials were accumulated, the step size was halved, and later halved again. Each staircase proceeded until 16 reversals (changes in the rank order of the two stimuli) were obtained at the smallest staircase stepsize

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(0.38 dB). In fact, two independent staircases were randomly interwoven for each stimulus pair as a check on convergence. The levels at which the last ten reversals occurred were then averaged to estimate the point of subjective equality. The record of a typical staircase is shown in Fig. 7. DOGs centered at  $1/2$ , 1, 2, and 4 cycles/degree were taken two at a time to form all six unique pairs. Two observers, including the author, were recruited for testing on each condition.

Results for the contrast matching sessions are shown in Figs. 8a-f. In most cases, a single regression line, shown dashed, was found to provide an excellent fit to the data. Regression parameters are shown at the top of each figure. Almost all of the best-fitting lines have slopes close to one and intercepts close to zero. The sample standard deviation associated with nearly every point in Fig. 8 is approximately 0.11 log units, (about 1dB), putting the majority of points within one S.D. of a veridical match. In a number of cases, there appear to be small, consistent shifts, favouring one DOG or another. Subject D.W. in Fig. 8a and subject S.H. in Fig. 8f are the most prominent examples of this. However, neither effect is borne out by the other subject under the same conditions. It is not known at this time whether these data represent real individual differences or systematic measurement error. At thresholds

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for detection, (which are just below the lowest contrasts measured here), there is known to be a 10:1 difference in the just detectable contrasts of the two extremes in frequency, 1/2 and 4 c/d (Wilson and Bergen, 1979). In light of this, it was decided to keep the contrast of all DOGs fixed at the highest feasible contrast for the subsequent motion studies.

Contrast as defined in Eqn. (3) depends on the peak and mean luminances; the DOGs of Eqn.(1) all have the same mean (zero) and all peak at 1.0 (when  $x=0$ ), so as defined, they all have the same contrast. It has been suggested that the total energy of a figure is a factor in the correspondence process (Burt and Sperling, 1981). The total energy in a waveform is proportional to the integral of the square of its amplitude, which for DOGs makes it proportional to sigma. It would be possible to match the different sized DOGs for total energy, but at the cost of mismatching their contrasts. As a check on the possible effects of total energy differences as opposed to spatial dispersion of that energy (size) it was decided to run some "energy-matched" probe trials. If the correspondence process discriminates on the basis of size, then pairs of DOGs differing substantially in size should show the most pronounced effect. Therefore the pairs (1/2 - 2) and (1 - 4) were altered so that the 1/2 c/d DOG had the same total energy as the 2 c/d DOG and the 1c/d DOG had the same total energy as the 4 c/d DOG. Should



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differences in the likelihood of seeing motion in the (2 - 2) pair versus the (1/2 - 2) pair be attributable to only to the total energy discrepancy, no difference should be observed in between (2-2) and the energy-matched (1/2-2) pair.

The percentage of trials on which motion between the two figures is seen is the only dependent variable in the first study; although it is perhaps the most common dependent variable in simple apparent motion studies of this sort, it is also susceptible to factors that are not of special interest, particularly shifts in subject criterion. Several stratagems have been adopted to help stabilize the results. Instead of asking the subject to decide after a single pair of frames, the two DOGs were shown in alternation for 8 complete cycles; under optimal conditions, an object will appear to oscillate to and fro when presented in this manner. Deciding whether the figure moves consistently for the 8 cycles seems to be subjectively easier for subjects. Other authors have employed this cycling technique with success (e.g. Pantle & Picciano, 1976).

A second regimen employed for the stabilization of results was randomization of the stimulus pairs. The four different sizes combined to make 16 different pairs. Ideally, all 16 pairs would be randomly distributed over the entire period of a subject's participation. Practical

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limitations on the subject's time and on the ability to retrieve waveforms rapidly from the computer's storage allowed only a modest approximation of complete randomization. Stimuli from the 16 pairs of DOGs mentioned plus the two energy-matched probes were chosen 6 at a time without replacement. Each group of 6 was randomly permuted, presented to the subject, shuffled again, etc. to accumulate 10 trials for each of the six pairs. (The process continued until all of the pairs were chosen.)

A final step to improve the quality of our estimates is a most traditional one: run more trials. The randomization procedures described above were repeated 4 times for each subject. Using a different random sequence for each occasion assured that each pair of DOGs was seen in four different contexts. If being tested in a particular random group of 6 had an effect on reports, we would expect it to be neutralized by averaging over other random contexts. The total exposure for each pair thus comes to 40 separate trials.

A further factor that is known to affect the likelihood of reporting apparent motion is the interstimulus interval (ISI), the time between successive presentations. In the cycled display used here, this is both the time between the presentation of the first and second DOG and the

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time between the end of one cycle and the beginning of the next. This time parameter is known to affect the perception of apparent motion in the same fashion as spatial separation (Burt and Sperling, 1981; Caelli, 1981; Morgan, 1980). Rather than try to execute a very lengthy full factorial design, I decided to postpone manipulation of the distance parameter for a future study. Instead, three different ISIs were selected on the basis of published reports (Kolers, 1972) and preliminary observations, 67, 100 and 133 ms. These ISIs were likely to span unknown optimal ISI with the distance traveled fixed at 1 degree.

The apparatus used for the presentation of the patterns is the same as described for the contrast matching study. One DOG in each pair was presented on the line of fixation, while the other was centered one degree to the right. Subjects were asked to fixate on the central marker and press a button to start the display when ready. If the display appeared "to oscillate from left to right and back for most of the duration of the show", subjects were asked to indicate so by pressing a button; another button was reserved for all other cases, such as no motion or sporadic motion.

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Four subjects, including the author, completed the regimen described above at each of the three ISIs. Three of the subjects were naive to the purposes of the study, although all had participated in other studies of visual perception. Sessions were limited to approximately an hour to avoid undue fatigue. Subjects finished the entire program in about 2 weeks. The importance of careful fixation during presentation was stressed to subjects; eye movements can have effects on the appearance of apparent motion displays. Two of the subjects were emmetropes; the other two were fitted with corrections for myopia during all runs.

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Experiment One: Results and Analysis

The percentages of trials on which subjects reported motion are plotted in Figs. 10-13 for the four subjects respectively. Each graph within a given figure represents the results for all pairs of DOGs with a particular size DOG shown on fixation. So each panel of Fig. 10 shows data for subject S.A.H. at a particular center DOG with the frequency of the off-fixation DOG plotted along the abscissa. The filled figures (circle, square and triangle), represent points for the three different ISIs: 67, 100 & 133ms. respectively. Compiled data is shown in Appendix 2.

The reader may have noted that two percentages have have been plotted for every size pairing of DOGs, (A,B): one with A on fixation (A-B), and one with B on fixation (B-A). It is also true that the data could be replotted with the frequency of the fixated DOG along the abscissa instead of the frequency of the off-fixation DOG, as it is shown now. There is little evidence in this data that the fixation of one or the other member of a pair makes a difference in the likelihood of seeing motion between them. This is consistent with the results of Kolers (1977). The data for subject S.T. in the 100ms ISI condition has been plotted both ways in Fig.

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14 for comparison; dotted lines represent data with the center frequency on the abscissa, while represent data with the off-fixation frequency on the abscissa as in Fig. 12. The two most notable exceptions are the (1/2--1) pairs of subjects S.H. and A.S. The percentages for the 1/2 on-fixation pair and the complementary pair (1/2 off-fixation) percentages for these two subjects are plotted in Fig. 15. The differences are unexpectedly large for both subjects. It is also true that the discrepancies are in opposite directions for the two subjects; S.H. is more likely to see motion with the larger DOG on fixation, while A.S. is more likely to see motion with it off fixation. The fact that these discrepancies are in opposite directions and that the other two subjects fail to show such differences, relegates this effect to the status of a curiosity until a much larger number of subjects have been run. This is not, however, the most interesting feature of the data.

The intriguing aspect of the data in Figs. 10-13 is the preference shown for movement between like-sized DOGs. This may be made more obvious by averaging the results over the four subjects. Since the question of fixational symmetry is, momentarily, not an issue, we may also average the complementary pairs, i.e. pairs that differ only in which DOG is on fixation. The results of these manipulations are shown in Fig. 16. Each curve peaks where the size

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difference is zero, and falls off monotonically elsewhere. The preferences for (1-1) over (1-2) and for (2-2) over (2-1) are both modest, but significant at the .05 level using Wilcoxon's test for matched samples ( $T=14, n=12$ ). This preference for like-sized DOCs is consistent with the notion advanced earlier that size information is pertinent to the correspondence process.

The probe trials that were energy matched instead of contrast matched are shown as open symbols on Figs. 10-13; the same ISI legend as used on the filled symbols pertains. The energy-matched DOC pairs are (1/2-2) and (1-4) as mentioned earlier. The total energy hypothesis would have that equalizing the total energy would render (1/2-2) equivalent to the (2-2) pair in sponsoring motion, and similarly equate the (1-4) pair to the (4-4) pair. A quick glance at the data will reveal that this position is untenable. While the (2-2) and (4-4) pairs generate motion reports nearly 100% of the time, the energy matched pairs are as depressed as their contrast matched equivalents. The single exception is the (1-4) energy matched pair at 133ms ISI for subject T.E.; at 100%, it fulfills the predictions of the total energy hypothesis; but, as a single point it holds little weight. Although energy factors may play some role in the correspondence process, they are not sufficient to explain the effects demonstrated here.

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The question of ISI effects in these data is somewhat more difficult to answer. At first glance (Fig. 16) they would appear to be negligible. Indeed they are modest, but there is some orderliness to their variation. Consider the longest ISI (133ms, triangle) and the shortest ISI (67ms., circle). It appears that where lower frequencies are involved, both on and off fixation, the circles dominate the triangles; and, conversely, where higher frequencies are involved, the triangles dominate the circles. There are exceptions, of course. This sort of categorical interaction is difficult to quantify without embracing dubious assumptions. I will present a scheme that will hopefully be intuitively pleasing despite being somewhat arbitrary. The key to the scheme is to code each pair of DCGs by the sum of their frequencies; instead of using actual centre frequencies, the four values have been integerized (by taking 1 plus log base 2 of the frequency) to the values 0 through 3 for 1/2 through 4 respectively. The difference between percent seen at the shortest ISI and percent seen at the longest ISI (circles-triangles) will then hopefully be predicted by the sum of the coded frequencies. Fig. 17 shows a scattergram with these sixteen points from the composite data of Fig. 16. On the vertical axis is the difference in percentage seen at the shortest ISI and the longest ISI; on the horizontal axis is sum of the coded



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frequencies for each DOG in the pair. The visible tendency of the points to cluster around a line of negative slope suggests that the frequency codes are not such bad predictors. A rank order correlation coefficient, Kendall's tau, for this set is  $-0.78$ , a significant value at the  $0.01$  level.

This can be seen as partial confirmation of the observation that at the shortest ISI the pairs with larger DOGs were more likely to be seen in motion than pairs with smaller DOGs, while at the longest ISI, the inverse relation holds. It is possible that this reflects differing temporal properties of simple motion detecting units tuned to different sizes. The slope of Fig. 17 would, in this view, indicate that mechanisms arranging the correspondence of smaller DOGs had a longer time constant than those responsible for the correspondence of larger DOGs. Given the small absolute magnitude of the observed effect, these speculations must await a more thorough manipulation of stimulus timing before being seriously considered.

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The evidence presented so far suggests that the closer DOGs are in size the more likely they are to be seen in apparent motion. The tentative explanation is that whatever mechanism is responsible for developing the correspondence for motion perception uses size information to help determine whether objects should be connected in motion. Ullman (1979) has suggested that the various attributes of object pairs such as colors, orientation, size, spatial and temporal displacement are somehow aggregated in a single metric, dubbed "affinity". This is taken as an index of goodness of fit between any pair of primitives of the correspondence process. The affinity of a given pair does not take into regard the other possible pairs that either of its members may participate in. Rather, it reflects that pair's quality in isolation. Affinity is an aggregate in the sense that changes made in one attribute of the pair, say distance, can be made up for by changes in another attribute, say ISI. The experimental results obtained here would, in this view, reflect changes in the affinity of pairs due to size discrepancy.

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In complex moving scenes, the affinity associated with a given pair is not the only thing that determines whether that pair is ultimately seen in motion. In all but the simplest cases, a single primitive element may participate in many pairs; it is the correspondence process that must decide which of the many possible mappings will be selected. Ullman's thesis is basically that the mapping which globally maximizes affinity is selected. Various constraints on the process are built-in, according to Ullman's scheme, such as a preference for mappings that are both one to one and onto as opposed to those that are merely one to one. Nonetheless, the raw materials of the correspondence process are the affinities of the pairs.

If the measurements with isolated pairs of DOGs, made in Experiment One, do indeed reflect the underlying affinities of the pairs, then they may be useful in predicting the results of more complex displays using the same elements. A strategy suggested by Ullman for examining affinity differences is to pit two (or more) pairs directly against each other. This is the "competing paths" scheme mentioned before. As illustrated in Fig. 18, it involves the presentation of a single object (A) in the first frame, and after a preset time, the presentation of two flanking objects (B and C). Such an arrangement will hereafter be

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referred to as the triple, (B-A-C). The two frames can, of course, be alternated as was done in the previous experiment. Several different percepts might result from such a display. (A) might "split" and move to both (B) and (C). (A) might move to (B) exclusively or (A) might move to (C) exclusively. Finally, there may be no motion perceived at all. When the spacing and timing are well adjusted and all objects are identical, the predominant percept is splitting (Kolars, 1972; Ullman, 1979). In cases where the affinity of one of the pairs, e.g. (A-C) exceeds that of the other (A-B), it is expected to draw more reports of motion since cases that would have been labeled "splitting" would then be (A-C) alone.

The configuration described above is the essence of the second experiment I have conducted. The objects (A, B and C) referred to in Fig. 18 were drawn from the kennel of DOGs used in the last experiment. All possible pairs had been measured in isolation previously, so it was decided to follow up by considering all possible triples for this "3-DOG" study; this comprises 64 different triples. One ISI was used throughout, 100ms, the average value of the three previously employed. The ISI was not manipulated because of the large number of conditions already defined, and because of its general impotence over the range previously investigated. The stimuli were arranged such that the central DOG peaked on

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fixation, as in the last experiment, and the two flanking DOGs peaked 1 degree of visual angle on either side of fixation. The two frames, central and flanking DOGs, alternated for 8 cycles.

This display system is as described for the last experiment, but two additional buttons were made available for subject responses. Subjects were instructed to press a "Split" button if both flanking objects appeared to oscillate from center to side for most of the duration of the show. If only one of the flanking DOGs satisfied this criterion, the subject was to press either a "Left" or "Right" button indicating which side offered the motion. Finally, if no consistent motion was observed, a "None" button was to be pressed. Stimulus presentations were randomized as before so each subject saw each triple 40 times.

Three subjects from the previous experiment served again in this one; subject T.B. was unable to continue, so another subject (N.L.) was recruited and run on the initial experiment, but in the 100ms. ISI condition only. Results from this catch-up run are shown in Appendix 2, and are in general agreement with the data from the other subjects.

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The results from the 3-DOG experiment were tabulated and are available in Appendix 3. These data are displayed graphically in Figs. 19-22. Each figure shows the results for all triples with a particular size DOG at their center (corresponding to object (A) in Fig. 18). Fig. 19 is for the  $1/2$  c/d centers, Fig. 20 for  $1c/d$  centers, Fig. 21 for  $2c/d$  centers and Fig. 22 for  $4c/d$  centers. Figure subscripts a..d index subjects S.H., A.S., S.T. and N.L.

For a given subject and center frequency, there are sixteen individual bar graphs, arranged on a square matrix. The vertical axis of the matrix indicates the spatial frequency of the left-hand DOG in the triple; the horizontal axis of the matrix indicates the frequency of the right-hand DOG in the triple. From left to right and top to bottom, these axes range from  $1/2$  to 4. Each of the 16 separate bar graphs shows three bars. The left-most bar indicates the relative frequency (in percent) of the reports of motion exclusively in the left-hand pair. Similarly the right-most bar indicates the frequency of reports of motion exclusively in the right-hand pair. The central bar indicates the percentage of reports indicating motion in both pairs

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simultaneously, i.e. splitting. Tic marks on the vertical axes place 50% and 100%. Since subjects are forced to make a response on each trial, percentages not accounted for by the three bars are taken up by the "no motion" response category. To help in identifying the triple associated with a particular bar graph, the three DOG center frequencies in the triple, (left, center and right), are printed over the tops of the bars from left to right.

This plotting scheme may seem confusing at first, but it does offer certain advantages over most alternatives. By means of an example, suppose we wished to know for subject S.H. how well DOG pair (1-1) did when pair (1-2) was offered as an alternative. Since both pairs have a 1c/d DOG on fixation, the information required will be in the second figure, Fig.20. Subject S.H. is represented in Fig.20a. The particular match of pairs specified actually appears twice: (1-1-2) and (2-1-1). The indices will help you find them at the second row from the top in the third column and in the third row from the top in the second column respectively. Mirror image triples like these always lie across the negative diagonal from each other. The first bar graph mentioned, (1-1-2), shows that about 60% of the time, the subject reported seeing motion only between 1 and 1; about 30% of the time, he reported seeing splitting-motion in both pairs (1-1) and (1-2). Motion exclusively between 1 and

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2 was reported less than 10% of the time. If S.H. were an ideal observer, the other bar graph would be a mirror image of the one already described. Actually, motion between 1 and 2 was reported less often in the (2-1-1) cases. Motion between 1 and 1 was seen just as frequently as in the (1-1-2) case, as though (1-1) motion reports which were included in the splitting category migrated to the (1-1) exclusive camp.

A casual examination of the "distribution of mass" on Figs. 19-22 will reveal some of the grosser agreements of the data with the predictions advanced. Fig. 19 shows results for triples with 1/2 c/d centers. As expected, most of the motion reports were issued along the top and the left flank, where other large DOGS were present in the triples. Curiously, all subjects seemed to see less motion when all three members of a triple were 1/2 c/d. This holds true vacuously for subject N.L. who reported almost no motion at all for triples with 1/2 c/d centers. (Fig. 19d). At the other end of the spectrum, triples with 4c/d DOG centers (Fig. 22) show the converse effect. Most of the mass hugs the bottom and right flank where the smaller DOGs are assured to be in the triples. Figs. 20 and 21 representing the 1c/d and 2c/d center triples respectively, show mass distributed closer to the center of the matrix, as befits their intermediate size status. If you blur your eyes, you might even see the 1c/d centered matrix curving slightly to the



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upper left corner and the 2c/d centered matrix curving to the lower right corner.

In order to check the predictions involved in the affinity hypothesis more rigorously, an index of preference needs to be chosen that is applicable to data from both experiments. There are many ways to define such a measure, and it is not clear a priori which should be chosen. However, certain properties are desirable: it should be continuous and well defined at the boundaries (where one pair is preferred completely to the other); it should take into account the magnitude of the indifference shown towards one pair versus another, as expressed by splitting or null reports; finally, hypothetical data from both experiments ought to cover the same range. Eqn. (4) is a simple metric that meets these requirements. L and R are defined for the 3-DOG study as the percentages of reports of exclusive motion in the left and in the right pairs respectively. In the data of the original experiment, L and R are defined as the percentages of trials on which motion was reported for each of the two pairs that were joined in the latter experiment as a triple.

$$(4) \quad p = 0.5 * ( 1 + (L-R)/100 )$$

Harkening back to the example in Fig. 18, preference in the

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triple (B-A-C) would be computed from the 3-DOG data by using the percentages of left-only and right-only reports as L and R in Eqn. (4). That preference would be predicted, hopefully, by the percent seen in motion data from the first experiment for pair (A-B) as L and pair (A-C) as R. Both L and R are restricted to the range [0,100]. If all reports of motion on a given triple, (B-A-C) are left-exclusive, L will be 100, R will be zero and so preference  $p=1$ . If, on the other hand, all reports are exclusively to the right, L will be zero, R will be 100 and so  $p=0$ . If L is equal to R, then  $p=0.5$ . Although splitting and no-motion responses are not explicitly referenced in Eqn. (4), they are not without effect on p. If considerable percentages of reports are either of splitting or no-motion or both, the maximum absolute difference between L and R is likewise restricted; smaller differences will produce less deviation in p from 0.5.

The calculated preferences from both experiments are tabulated in Appendix 4. A scattergram (Fig. 23) shows the predicted against the observed preferences for the entire collection of data (256 pts = 4subjects \* 64 triples). Linear regression might be foolhardy; however, order statistics are applicable. Kendall's tau is 0.35 for this data set. With 256 points, this is a highly significant figure ( $p<0.01$ ). The hypothesis that there is only chance

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connection between the two variables must be rejected. As with other correlation coefficients, however, a tau of 0.35 is not exactly a howling success, so the question "Why wasn't it better?" is well worth asking.

First, there is some variation from subject to subject in terms of the value of our predictor. Table 1 shows tau for each of the four subjects, computed separately.

TABLE 1  
Rank Order Correlation of Preference by Subject

<u>Subject</u>	<u>Tau</u>
S.H.	0.37
A.S.	0.58
S.T.	0.31
N.L.	0.18

The relatively low tau associated with subject N.L. can be attributed in large part to the extremely depressed responses for triples with 1/2 and 4 c/d centers. Table 2 shows that overall, these two extremes of center frequency showed the highest correlation, while 2 c/d centers showed the lowest.

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TABLE 2  
Rank Order Correlation of Preference by Center Frequency

<u>Center Freq.</u>	<u>Tau</u>
1/2	0.60
1	0.34
2	0.12
4	0.46

Where else do the predictions break down? Fig. 20 (1c/d centers) shows some of the story. For all subjects but A.S., the triples (1/2-1-1) (1/2-1-2) and their inverses generated reports inconsistent with what was expected from the earlier behaviour of the pairs (1-1/2), (1-1) and (1-2). In particular, it was expected that the (1-1) pair in the (1/2-1-1) triple would be seen in motion more often than the (1-1/2) pair; quite the contrary seems to be true for these subjects. Similarly the (1-2) pair was previously judged to be more consistently seen in motion than the (1-1/2) pair; again the (1-1/2) pair dominates. Subject A.S. also showed this order in the 3-DOG data; her distinction is that these results are more consistent with her performance in the first experiment. A few internal irregularities can also be spotted in these figures. Most notably, subject S.H. seemed to prefer (1-1/2) motion in triple (1/2-1-1) but (1-1) motion in triple (1-1-1/2); if he simply preferred seeing things

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moving to the left, this asymmetry should have asserted itself elsewhere, but this is not evident.

When one considers triples with  $2c/d$  centers (Fig. 21), the picture becomes even more confusing. With all subjects, a major discrepancy was the general lack of motion reports between (2-4) pairs in six of the seven triples containing them (all but 4-2-4). But the (2-4) pair engendered motion reports quite reliably when shown in isolation. With subjects S.T. and A.S. (Figs. 21b and c), the (2-1/2) pair showed a curious strength, inconsistent with that demonstrated in the previous experiment. (This is evident in the left-most columns of the two figures.) The least explicable aspect of the data is the asymmetry shown in Fig 21b (subject A.S.). The pair (2-1) dominates in the triple (1/2-2-1) while the pair (2-1/2) has the edge in symmetrical triple (1-2-1/2). The case is similar for the triple (1/2-2-2) and its mirror image (2-2-1/2): (2-2) is preferred in the former and (2-1/2) in the latter. Triples (1/2-2-4) and (4-2-1/2) offer another example of this asymmetry. It is difficult to explain. A fixational error might alter the relative orders in this manner, but any such eccentricity would manifest itself on other pairs in a like manner since the triples were randomly interwoven. For example, in the same figure, the triples (1-2-4) and (4-2-1) ought to consist largely of (2-4) reports for the former and

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(2-1) reports for the latter; on the contrary, the (1-2) pair is more frequently seen in motion in both triples.

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A Model Relating the Two Experiments

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Ignoring for a moment the handful of internal inconsistencies with the 3-DOG data, a conclusion that can be drawn from reported motion in triples is that the perceived motion of one pair is not in general independent of the perceived motion of the other. It is possible to look down almost any row or column of the Figures 19-23 and observe variation in the reports pertaining to the same pair. In making the predictions from the pairs in isolation, however, independence was implicitly assumed. Motion in each pair of the triple was assumed to be an independent random process akin to the toss of a biased coin. Each triple would have two such coins. To stretch the analogy, if one coin turns up heads, and the other tails, then motion would be seen on one side only. If both coins turn up heads then splitting is reported; if both tosses produce tails, then no motion is reported. Knowing how one coin turns up on a given pair of tosses tells you nothing about how the other will turn up. Since, under our assumptions, the tosses are independent, the expected preference depends only on the biases of the two coins; supposedly we could measure those biases just as well by tossing the two coins separately as in the first experiment. Clearly this is not the case; the nature of the interaction between the two motion paths will have to be

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taken into account in order to improve on the predictions of the first experiment.

One hypothesis that has been advanced (Ullman, 1979) to explain the preference shown to one pair or another in situations such as the second experiment is that of competition. Put simply, if a given object could follow various different paths in an ambiguous situation, these alternatives will "compete"; the path offering the best overall features (e.g. maximal affinity, symmetry) will be seen most frequently and at the expense of the other paths. This postulates that paths interact in a system that on any given presentation must pick one path and exclude all the rest. Many perceptual phenomena are like this; dramatic examples are the bistable illusions such as the Necker cube and the "My Wife and My Mother-in-Law" cartoon (Boring, 1930; Attneave, 1971).

There are certain drawbacks to the application of this idea to the second experiment. The most obvious of these is that the perception of motion in the left and right pairs is not strictly exclusive. In some cases both are perceived at once, and in some cases neither is perceived. If one path were chosen to the strict exclusion of the other, triples like (2-2-2) ought to draw 50% left and 50% right reports instead of 100% splitting. However, it may be possible to



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sidestep this problem while preserving the notion of competition. I will advance a modest attempt to reconcile the pairwise percent-seen data of the first experiment with that of the second by means of a modified competition scheme.

The hypothetical quantity which will be the currency of competition shall be called "strength",  $S$ . Every pair (a-b) of objects that could potentially be seen in motion has associated with it a strength  $S(a,b)$ . Key to the derivation of any predictions about observers' reports is the admission that  $S$  is a random variable. This premise distinguishes strength from affinity as previously discussed; the latter quantity we may wish to associate with some parameter of the distribution of strength. The triples used in the second experiment can be considered as bearing two such variables:  $S(a,b)$  and  $S(a,c)$ . Suppose the difference between the two variables were the arbiter of observers' reports. If the absolute difference  $|S(a,b) - S(a,c)|$  is less than some constant,  $c$ , then one of two things will happen. If both strengths are small, no motion will be reported. If both strengths are large, then splitting will be reported. If the difference between the two strengths is greater than  $c$ , we will suppose that the pairs compete and the pair with the larger strength will alone be reported in motion. This is the essence of the interpretation of competition that will be considered. The shortcomings already apparent in the

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choosing between splitting and null responses I ask the reader to excuse in order that the question of relating the results of the two experiments in terms of preference alone might be addressed.

Ignoring for the moment the constant  $c$ , which we will assume is the same for all triples, the competition scheme described above bears considerable resemblance to Thurstone's law of comparative judgement (1927). The law was originally constructed to help explain choice probabilities in social psychology, but was defined in such generality that it has since been used for problems as diverse as perception and economics. The psychological variables that give rise to decisions between choices are dubbed "discriminal processes" in Thurstone's work. The law presumes the existence of means  $U(i)$ , standard deviations  $s(i)$  and a correlation coefficient  $r(i,j)$  the distributions of discriminial processes  $i$  and  $j$ . In all practical applications, the distributions are presumed to be normal, either by nature or through transformation done by the experimenter. The law can then be stated as :

$$(5) \quad U(i) - U(j) = z(i,j) \sqrt{s^2(i) + s^2(j) - 2r(i,j)s(i)s(j)}$$

where  $z(i,j)$  is the standardized score corresponding to the proportion of judgements observed where choice  $i$  is preferred to choice  $j$ .

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The statement of the law in (5) is the most powerful form that Thurstone considered; it is also the least tractable in terms of analysis of experimental data. Realizing this, Thurstone specified four additional forms of the law (known as cases II through V) each making stronger assumptions than the last. The simplest and most popular case is V, which assumes that the correlation coefficients are zero and that all standard deviations are equal. Case V assumptions and analyses are very similar to those commonly used in signal detection theory (Green & Swets, 1966).

In the current application, the discriminial processes are the strengths associated with each pair of stimuli. While it may be safe to assume, at least as a first approximation, that the strengths themselves are uncorrelated, the data suggests that the equal variance assumption would be foolhardy. This position is the one outlined for Case III of the law of comparative judgement:

$$(6) \quad U(i)-U(j) = z(i,j) \sqrt{s^2(i) + s^2(j)}$$

With minor modifications, this form of the law is readily amenable to application to real data. In order to estimate the parameters for each distribution, the data must be in the form of probabilities of choosing one pair over another. The

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best guess we have of such a probability is the preference,  $p$ , as previously defined. If our measurements of preference contained no error, the preference for an arbitrary triple (A-B-C) would always be one minus the preference measured for triple (C-B-A). The fact that preferences for some triples are not exactly the complements of their mirror images' preferences was reconciled by averaging. So if  $p=0.9$  for triple (2-1-4) and  $p=0.3$  for triple (4-1-2), the former was adjusted to 0.8 and the latter to 0.2. The actual estimation processes are fairly tedious and were carried out with the aid of a small computer. A derivation of the estimators is presented in Thurstone (1934) and in condensed format in Appendix 5. A noteworthy feature of the process is the use it makes of approximation; the approximations may be poor in data sets representing small numbers of stimuli or in which ceiling or floor effects are apparent.

The results of applying a Case III analysis to the data from the second experiment are shown in Table 3. The estimates of means and standard deviations are tabulated for the four subjects organized by center frequency. The 1/2 and 4c/d center triples for subject N.L. could not be modelled successfully in this manner, and so are omitted. This had been expected in light of the extreme suppression of responses she exhibited at these two extremes.

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Table 3

## Estimated Stimulus Distribution Parameters

On-Fixation DOG Frequency								
Freq.	Half		One		Two		Four	
Observer S.H.								
	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.
.5	0.94	0.69	1.0	1.85	-.56	2.92	-.24	0.16
1	0.07	1.43	1.09	0.96	0.63	0.54	-.30	2.23
2	-.46	1.37	-.23	0.28	0.57	0.30	0.42	1.60
4	-.55	0.52	-1.83	0.91	-.64	0.24	0.12	0.01
Observer S.T.								
	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.
.5	2.47	1.31	1.94	1.54	0.51	1.2	-.25	.35
1	0.42	0.54	0.66	0.70	0.74	0.84	-.21	1.06
2	-1.25	1.19	-.28	.57	0.07	0.97	0.43	2.25
4	-1.63	0.96	-2.32	1.2	-1.31	0.99	0.03	0.03
Observer A.S.								
	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.
.5	1.57	1.16	1.41	1.02	0.18	0.98	-.50	0.48
1	0.09	0.67	0.56	1.08	0.13	0.07	-.31	1.35
2	-.75	1.11	-.26	0.81	0.05	2.77	0.38	1.76
4	-.91	1.05	-1.71	1.09	-.35	0.18	0.43	0.41
Observer N.L.								
	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.
.5			0.22	2.11	-.58	2.11		
1			0.57	0.30	0.60	0.44		
2			-.18	1.27	0.40	1.15		
4			-.61	0.32	-.41	0.31		

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Estimates of both parameters have a considerable range. Consulting Table 3, means on the psychological scale vary from -2.32 to +2.47; standard deviations vary from 0.01 to 2.92. The parameters for S.H. for the pairs making up triples with 1c/d centers have been used to plot the strength density functions (Fig. 24). Some expected relationships between pairs are evident, such as the fact that the mean strength of pair (1-1) is greater than that of (1-2), which is in turn greater than that of (1-4). A less expected relationship is that between pairs (1-1) and (1-1/2): their means are very nearly equal. The strength of (1-1) should exceed that of (1-1/2) only about half of the time and vice versa. Furthermore, the mean strength of (1-1/2) exceeds that of (1-2) despite the fact that S.H. in Experiment 1 (see 10b) reported motion more frequently for (1-2) than (1-1/2).

One remaining task is to relate the estimates of distribution parameters to the results of the first experiment. How did observers in the first experiment decide whether or not to report motion when presented with a given pair of DOGs? One proposal, in keeping with the model just developed, is that they made their reports on the basis of the strength variable associated with the pair. The most mundane example of such a decision rule is to set a fixed threshold strength for reporting motion; observed strengths

### Size Factors in Apparent Motion A Model Relating the Two Experiments

greater than or equal to the threshold will elicit a motion report, while strengths less than threshold will not. Such a scheme is illustrated in Fig. 25. According to the model, strengths are distributed normally for all pairs, but with differing means and standard deviations. For each pair, estimates of the mean and standard deviation are available in Table 3. In order to calculate the probability that the strength of a given pair will exceed a fixed threshold, one simply has to convert the threshold to a standard normal deviation and consult any table of the normal distribution (Hays & Winkler, 1974) for the corresponding probability. Thus, if the threshold level used by the subjects in the first experiment were known, the probability of seeing motion for any given pair of DOGs could be predicted from the estimates of strength distribution obtained in the second experiment.

The selection of a threshold value is a problem in this line of reasoning. Without making further assumptions it cannot be extracted from the data in the second experiment. In particular, the modelling of the "splitting" and "null responses would need to be elaborated. Since this is not my interest in this paper, a simpler assumption about the threshold strength will be adopted. The means of the strength distributions, as a consequence of assumptions in the estimation procedure vary more or less symmetrically

Size Factors in Apparent Motion  
A Model Relating the Two Experiments

around zero, (the origin of the psychological scale was arbitrarily set there), so for the purpose of generating some typical predictions, the threshold strength will also be assumed as zero.

The predicted and observed percent-seen curves for all 16 pairs are shown in Figs. 26-29 for the four subjects individually. A set of curves showing the average of the four subjects is shown in Fig. 30. The peaked curves emerged from many of the predictions in more or less the right places. Another look at the rank order statistics for the 56 pairs of predicted and observed percentage-seen reveals a Kendall's tau of 0.44 (p.01), indicating that the predicted levels are not in such poor agreement. It is likely that this figure could be somewhat improved by adjusting the threshold levels with a best fit procedure, but this has not been attempted.

The simple model presented here should by no means be considered a complete explanation of the data collected in the second study. Rather, it was proposed with the intent of supporting the size-specificity findings of the first study. As such, it is notably incomplete and possibly inadequate in explaining the splitting and no-motion cases arising with triples. This aspect of the data was indeed more complicated than expected. Acknowledging these shortcomings however, the



Size Factors in Apparent Motion  
A Model Relating the Two Experiments

notion that the percent seen curves of the first experiment index some quantity relevant to perceived motion is generally bolstered by the second experiment in conjunction with a competition model.

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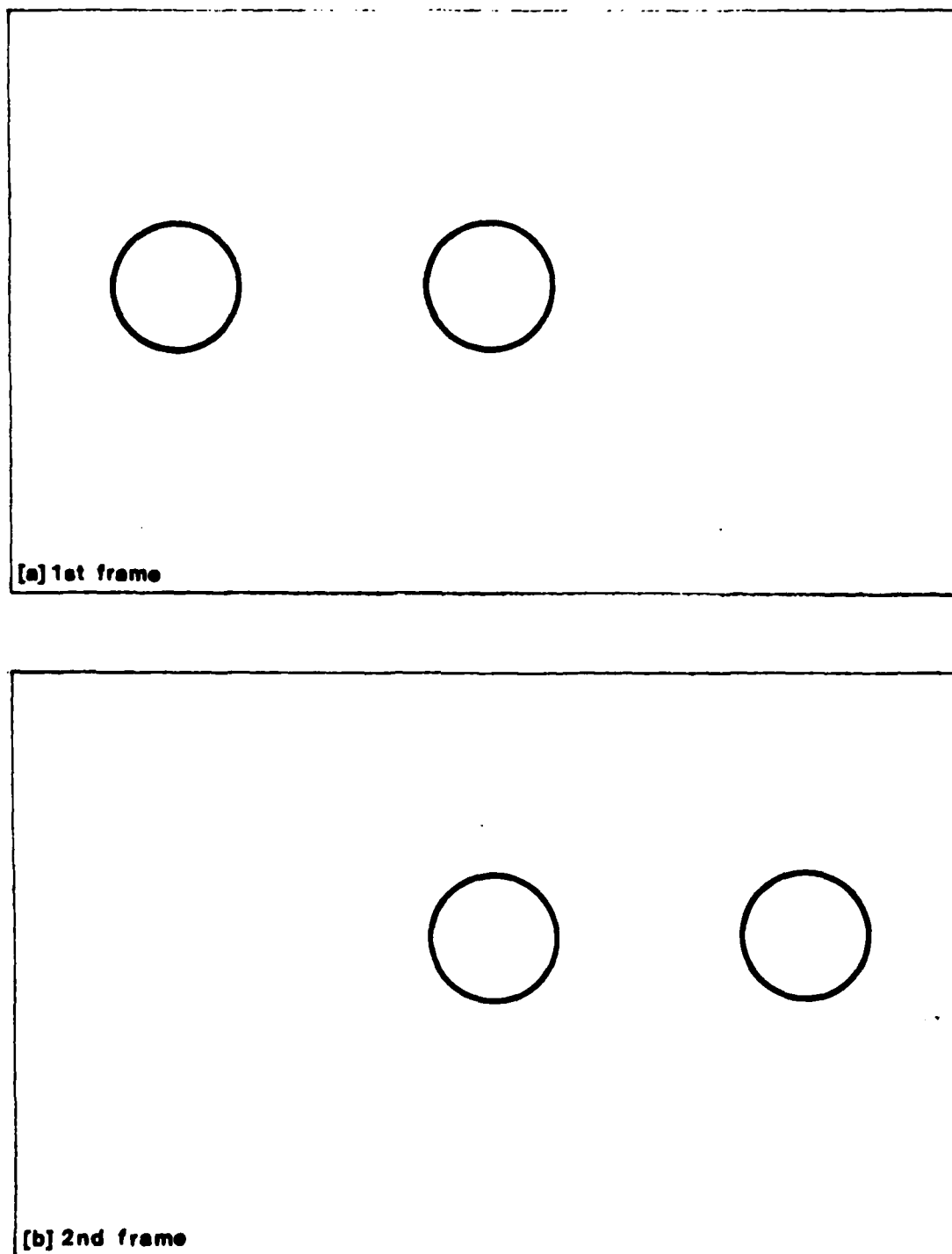


FIGURE 1 THE STIMULUS CONFIGURATION USED BY TERNUS (1926). WHEN THE FRAMES ARE PRESENTED ALTERNATELY, THE TWO DOTS APPEAR TO MOVE AS A GROUP, BACK AND FORTH.

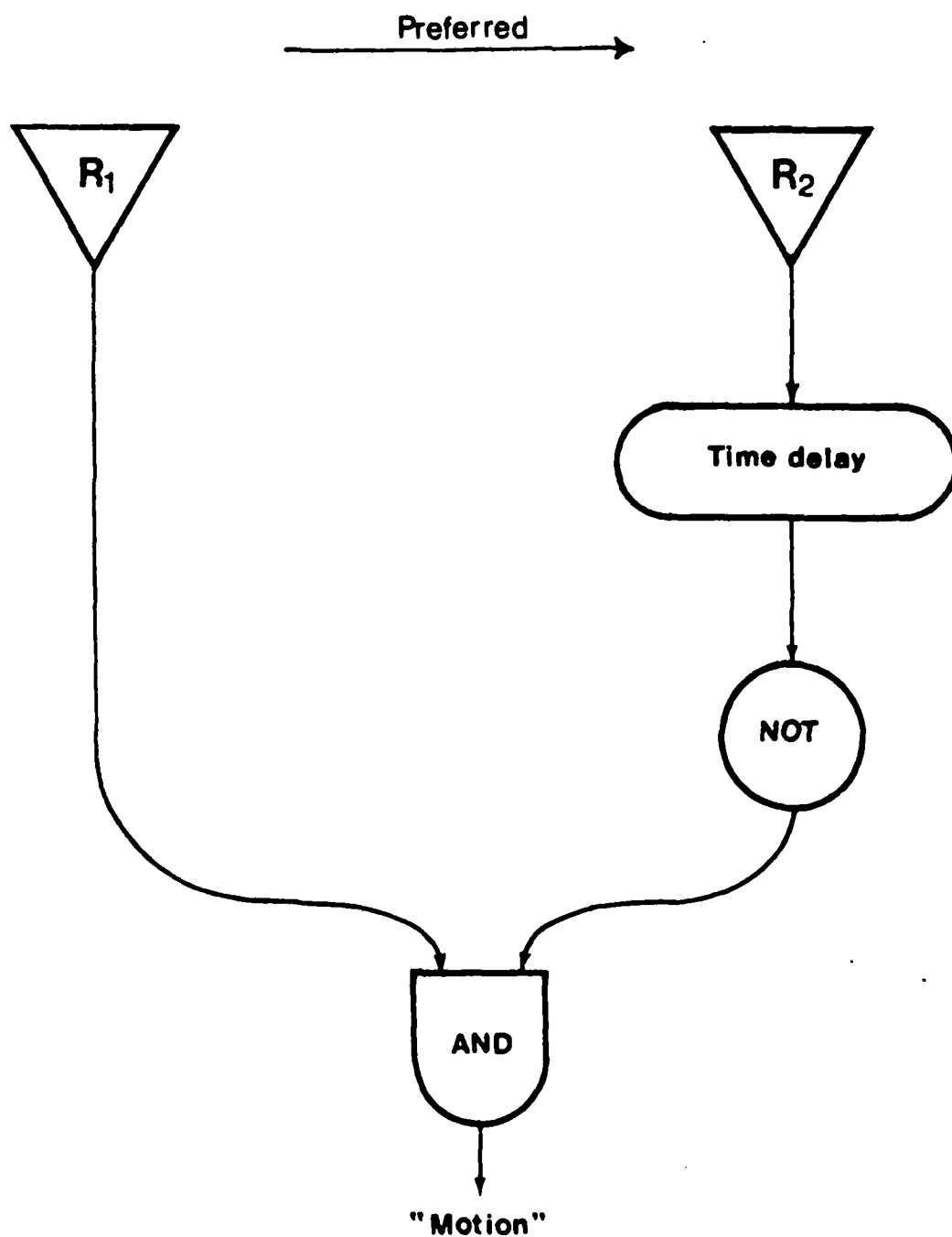


FIGURE 2 A SIMPLE MOTION DETECTION DEVICE AS PROPOSED BY BARLOW AND LEVICK (1965). OBJECTS MOVING IN THE PREFERRED DIRECTION ACTIVATE BOTH INPUTS OF THE AND GATE AT THE SAME TIME.

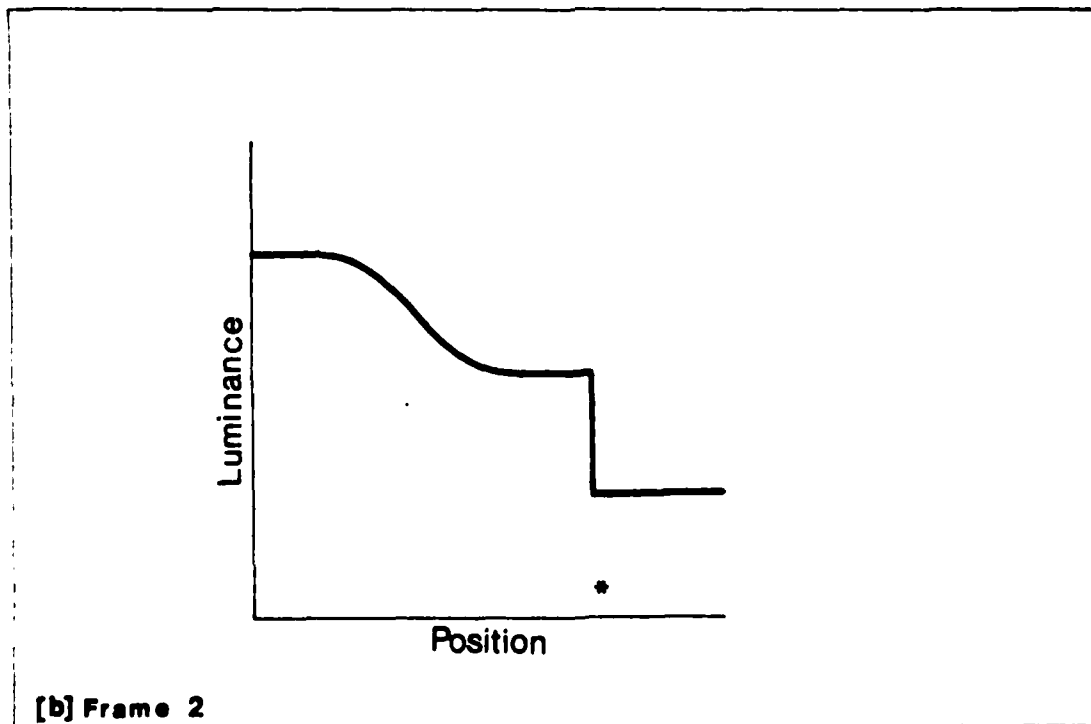
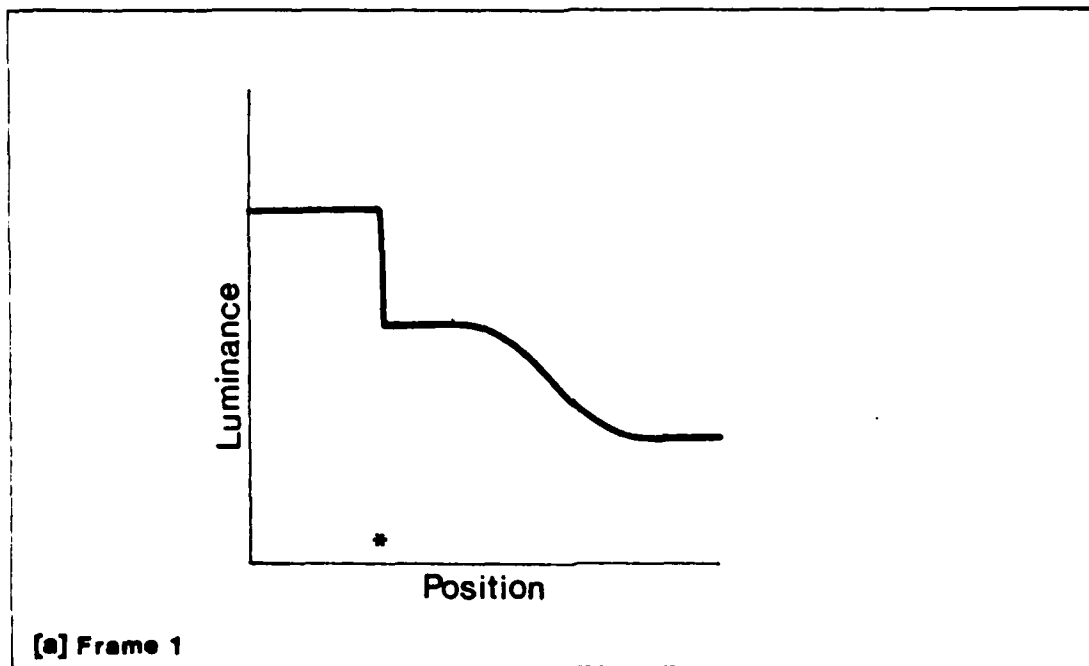


FIGURE 3. THE STIMULI USED BY ULLMAN (1979), DEPICTED BY THEIR LUMINANCE DISTRIBUTIONS. WHEN THE FRAMES ARE PRESENTED ALTERNATELY, MOTION IS SEEN BETWEEN THE SHARP EDGES (\*).

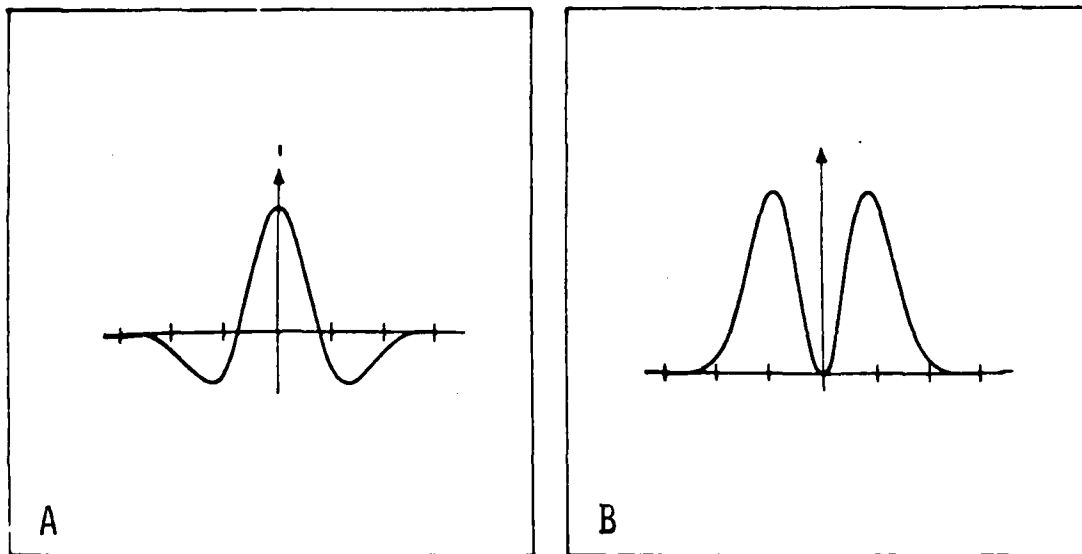


FIGURE 4 (A) DIFFERENCE OF GAUSSIANS (DOG) FUNCTION AND (B) ITS FOURIER TRANSFORM.

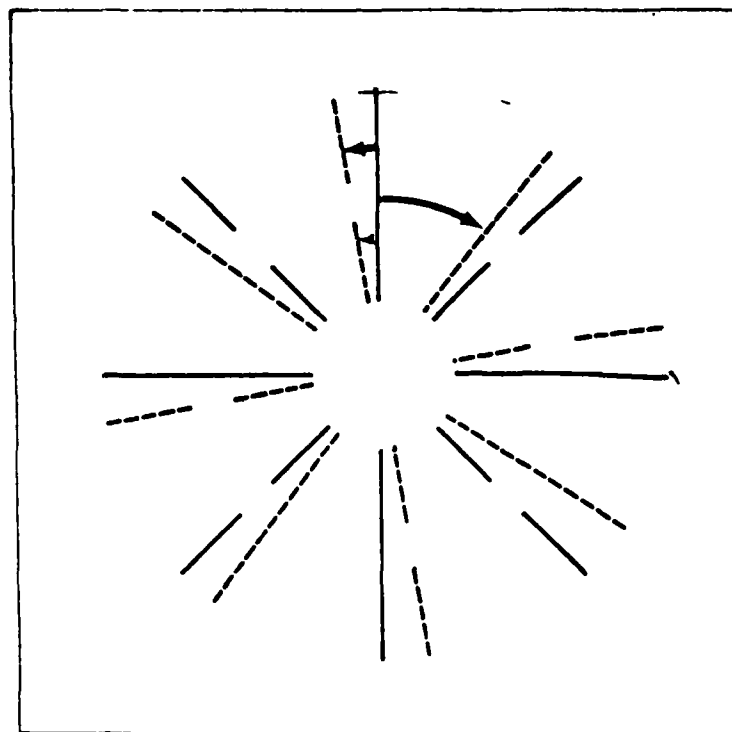


FIGURE 5 BROKEN WAGON WHEEL DISPLAY (ULLMAN, 1979) SOLID LINES SHOW THE FIRST FRAME, DASHED LINES THE SECOND. ARROWS INDICATE PERCEIVED DIRECTION OF MOTION FOR THE 3 SEGMENTS OF SPOKES



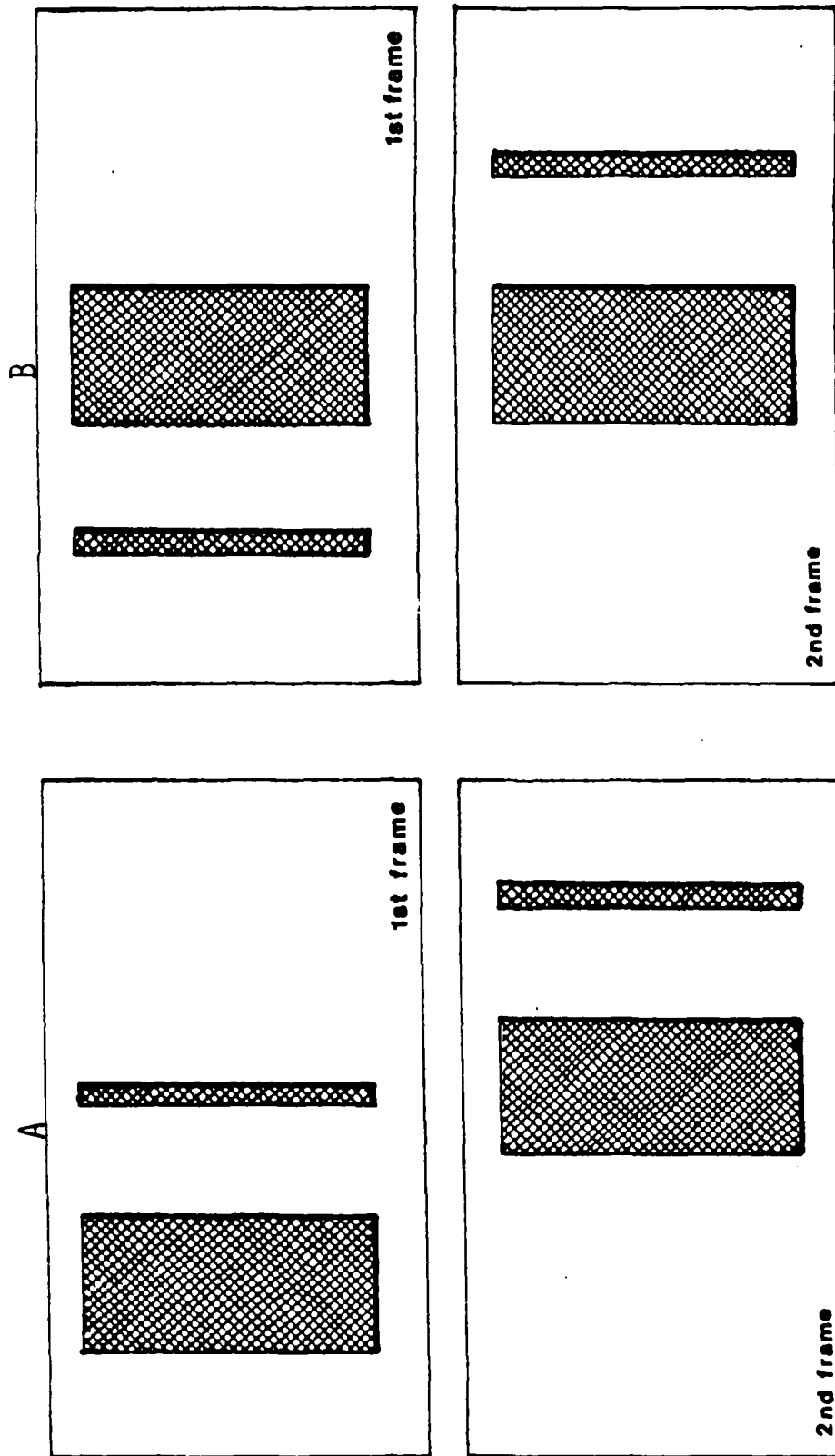


FIGURE 6 TERNUS DISPLAY WITH WIDE AND NARROW BARS. (A) RELATIVE POSITIONS OF THE BARS CONSTANT; BOTH BARS MOVE AS A GROUP. (B) RELATIVE POSITIONS REVERSED, ONLY THE SMALL BAR MOVES. AFTER FERNBERGER, (1934).

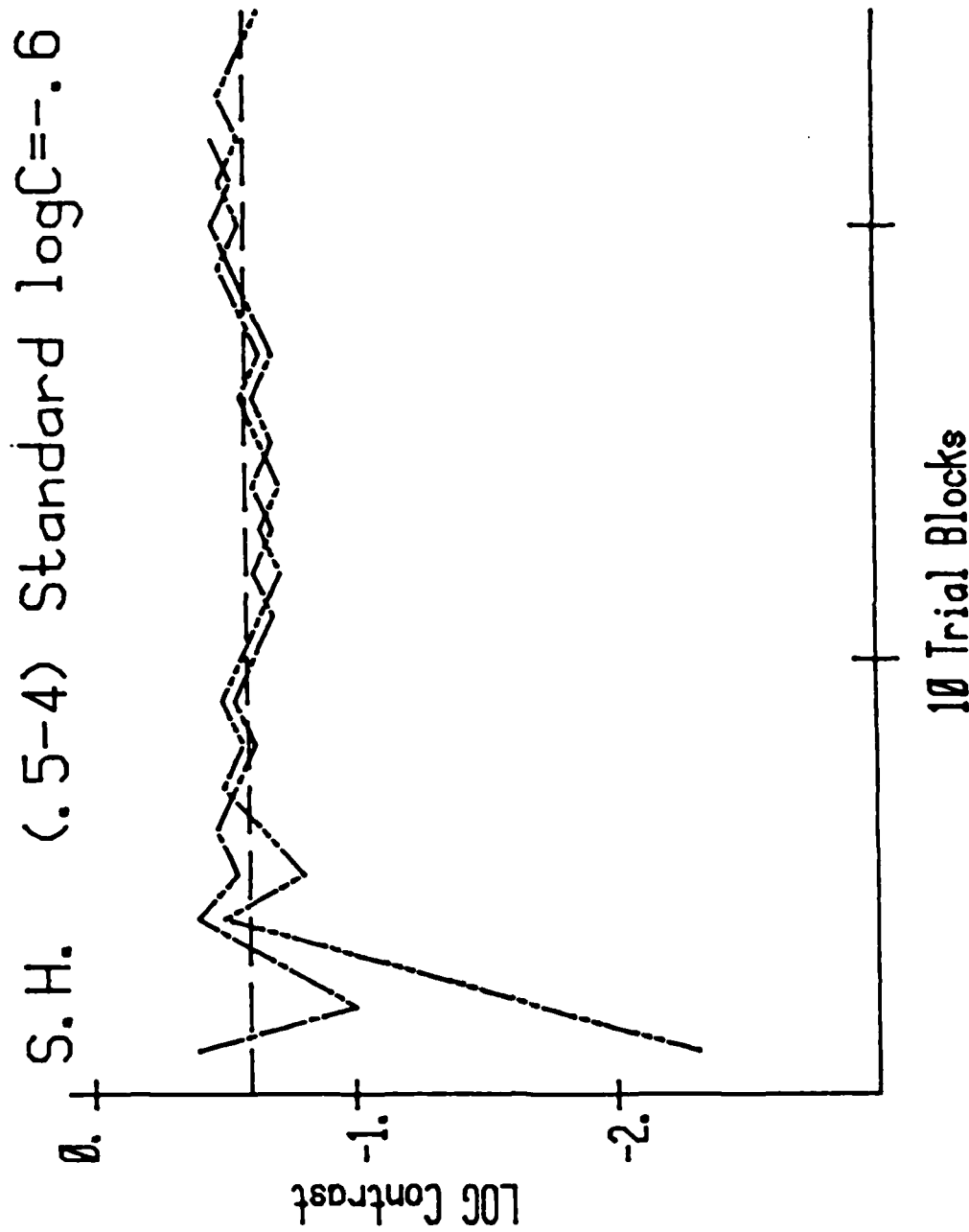


FIGURE 7 TYPICAL RECORD OF CONTRAST MATCHING SESSION: LOG CONTRAST OF MATCHING DOG AS A FUNCTION OF TRIALS FOR THE 2 STAIRCASES. DASHED LINE INDICATES STANDARD LEVEL.

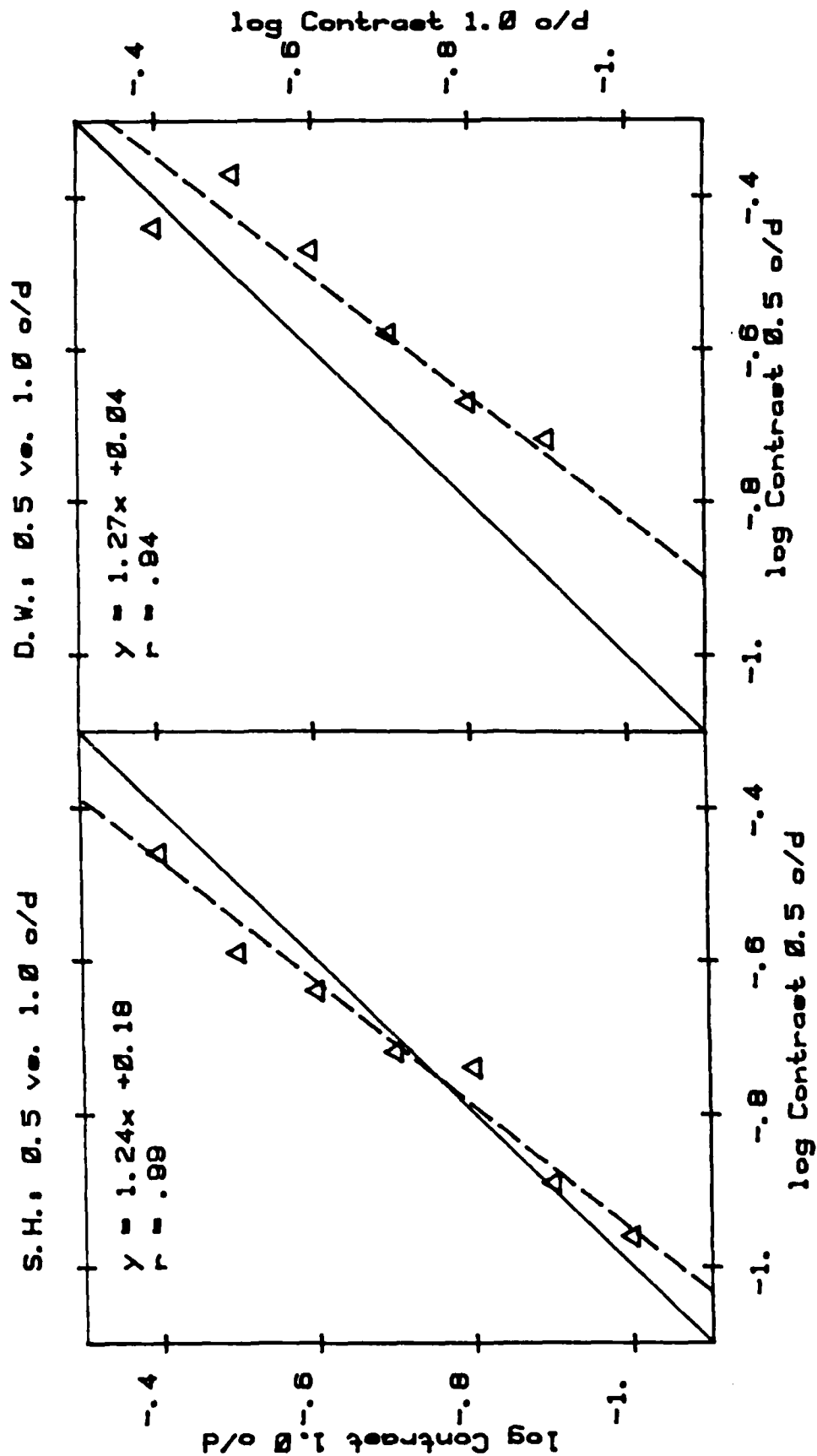


Figure 8A: contrast matching data for subjects  
 S.H. & D.W.: 0.5 versus 1.0 o/d DOGs

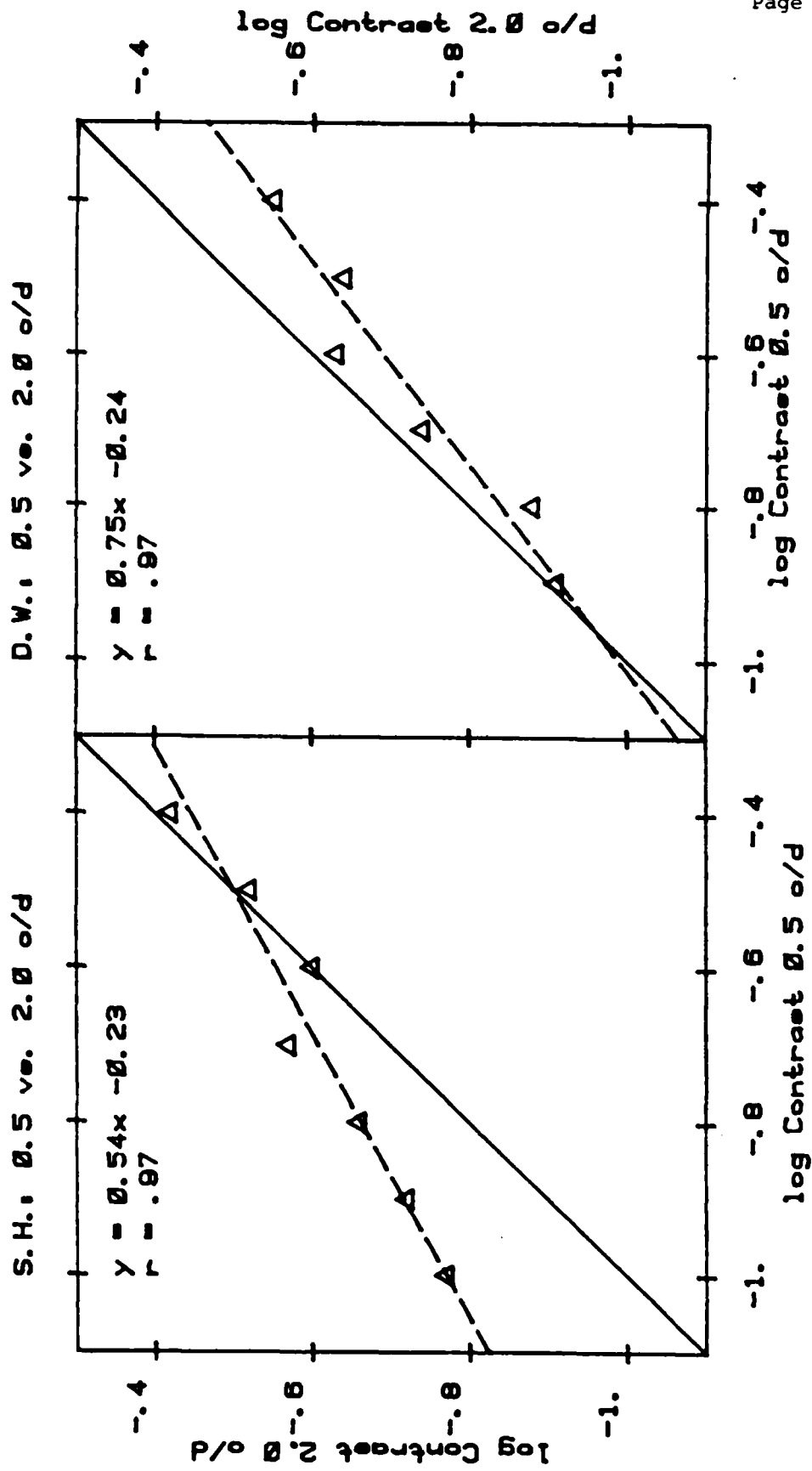


Figure 8B: contrast matching data for subjects  
S.H. & D.W.: 0.5 versus 2.0 o/d DOGs

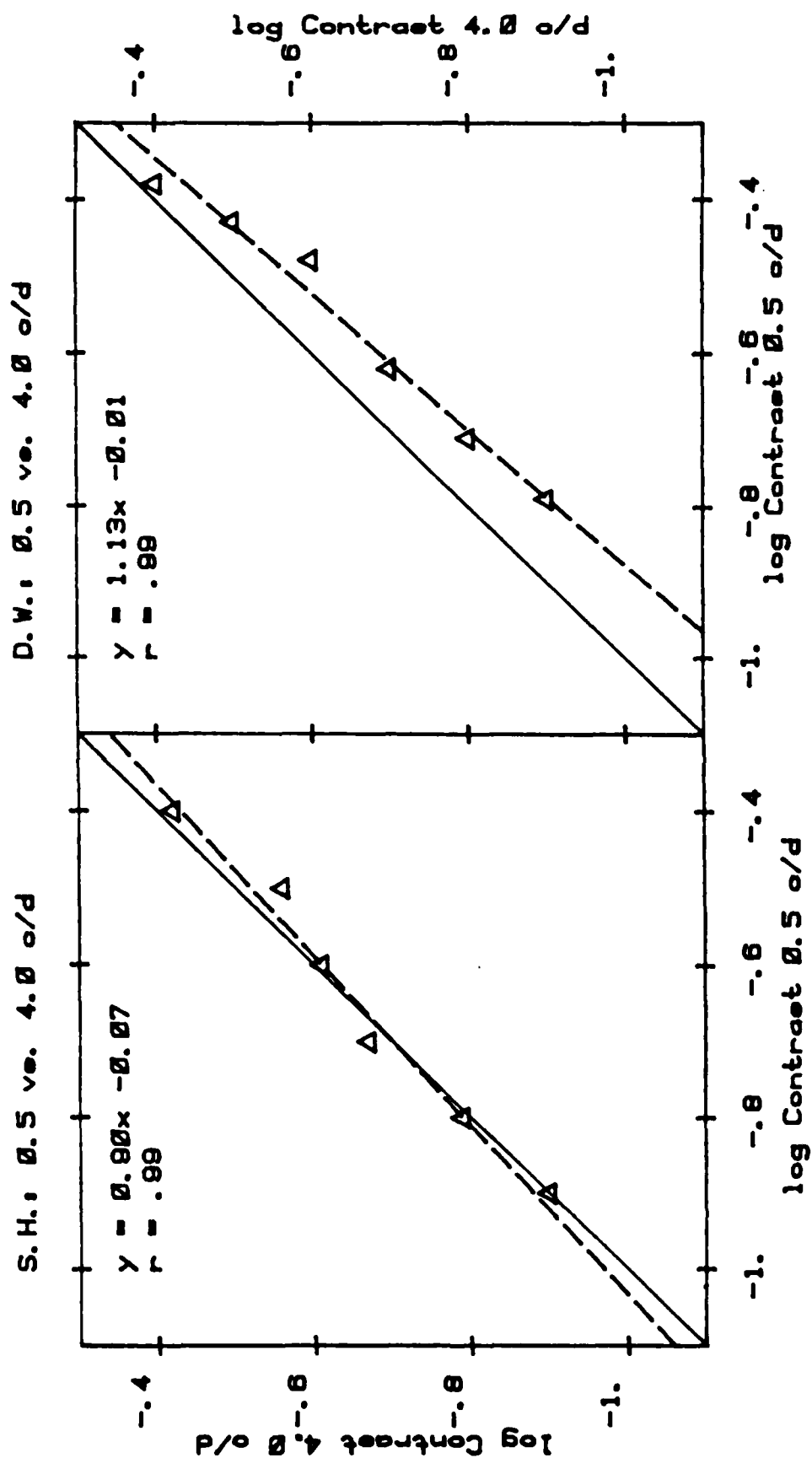


Figure 8C: contrast matching data for subjects  
 S.H. & D.W.: 0.5 versus 4.0 o/d DOGs

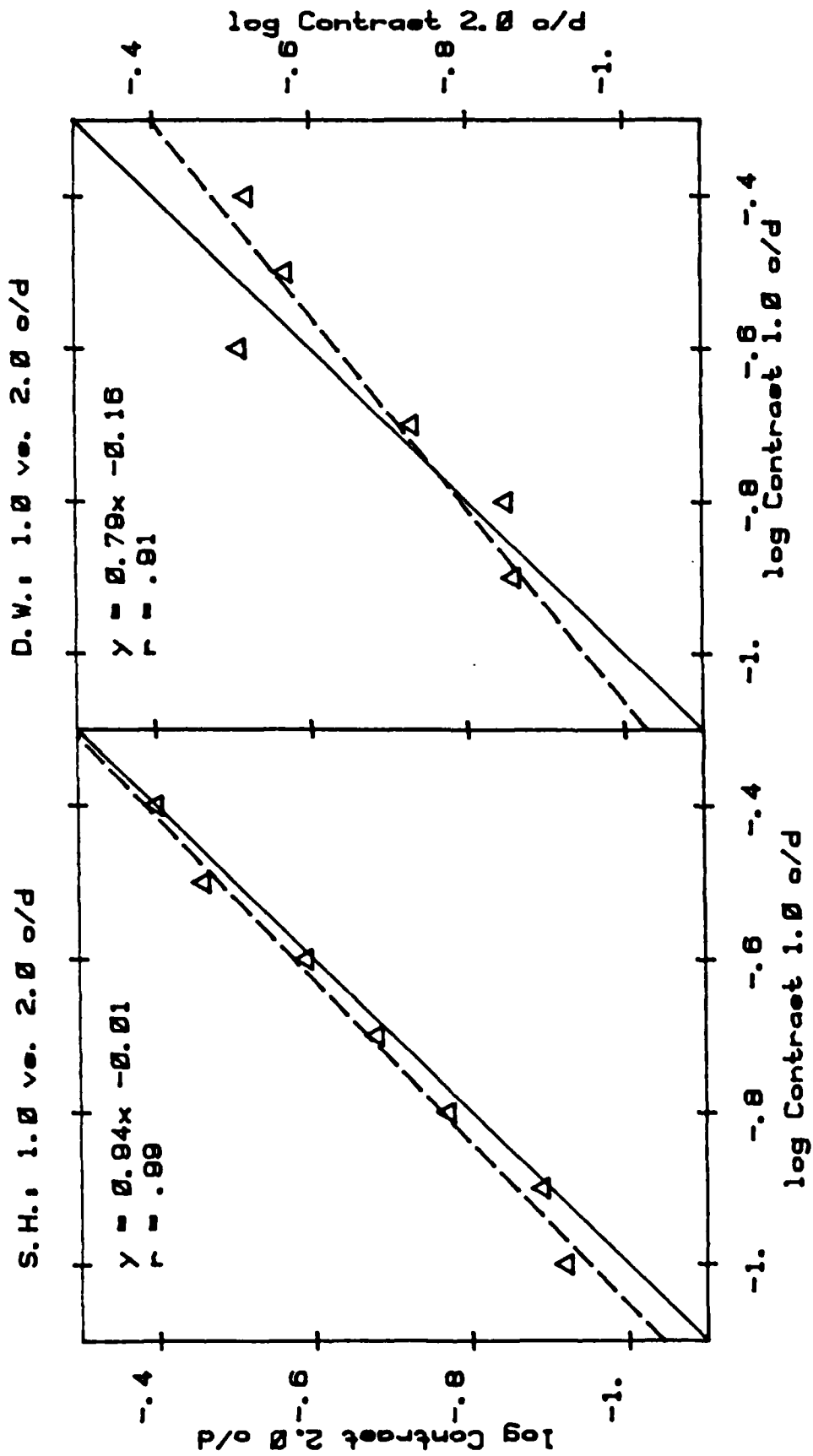


Figure 8D: contrast matching data for subjects  
S.H. & D.W.: 1.0 versus 2.0 o/d DOGs

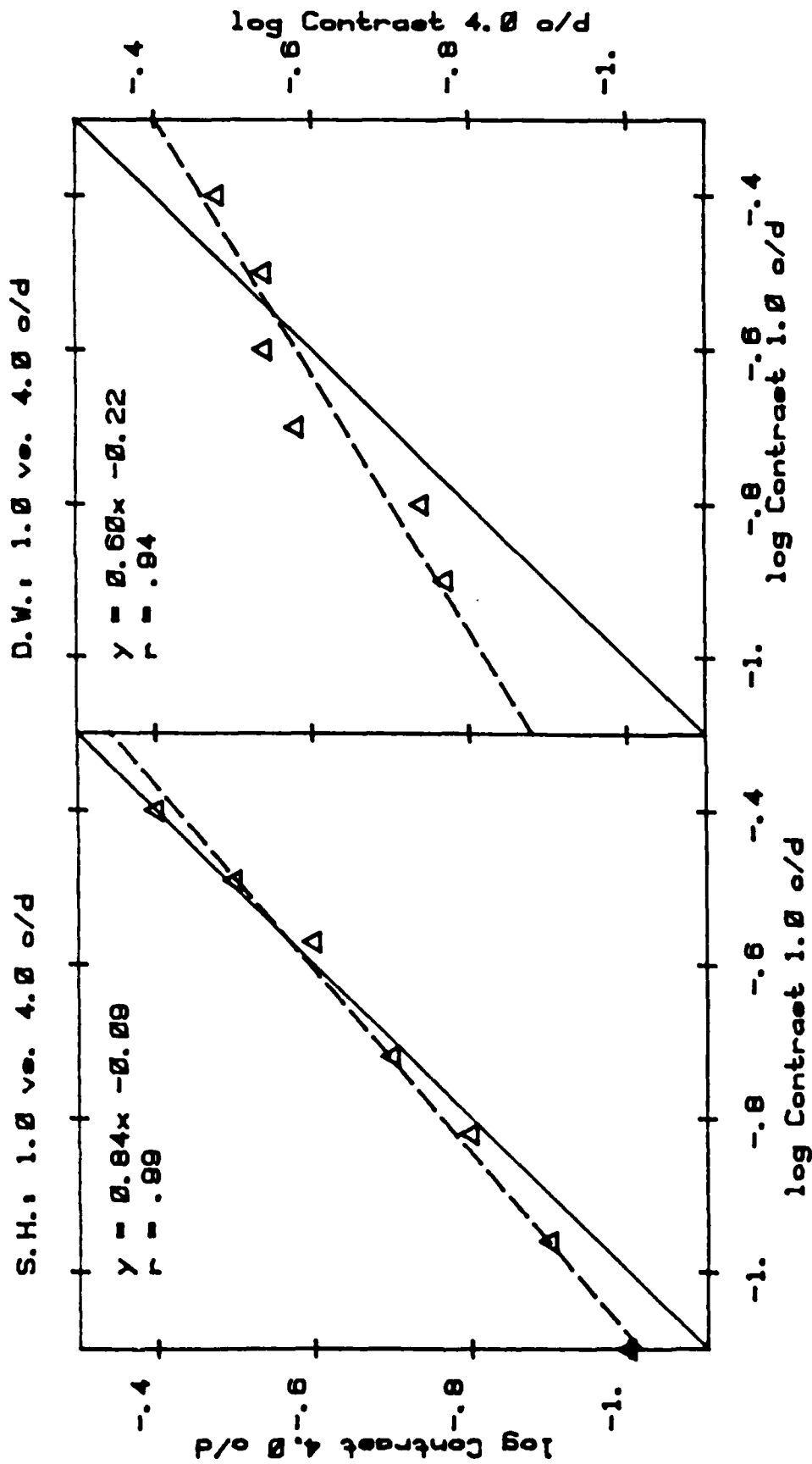


Figure 8E: contrast matching data for subjects  
 S.H. & D.W.: 1.0 versus 4.0 o/d DOGs

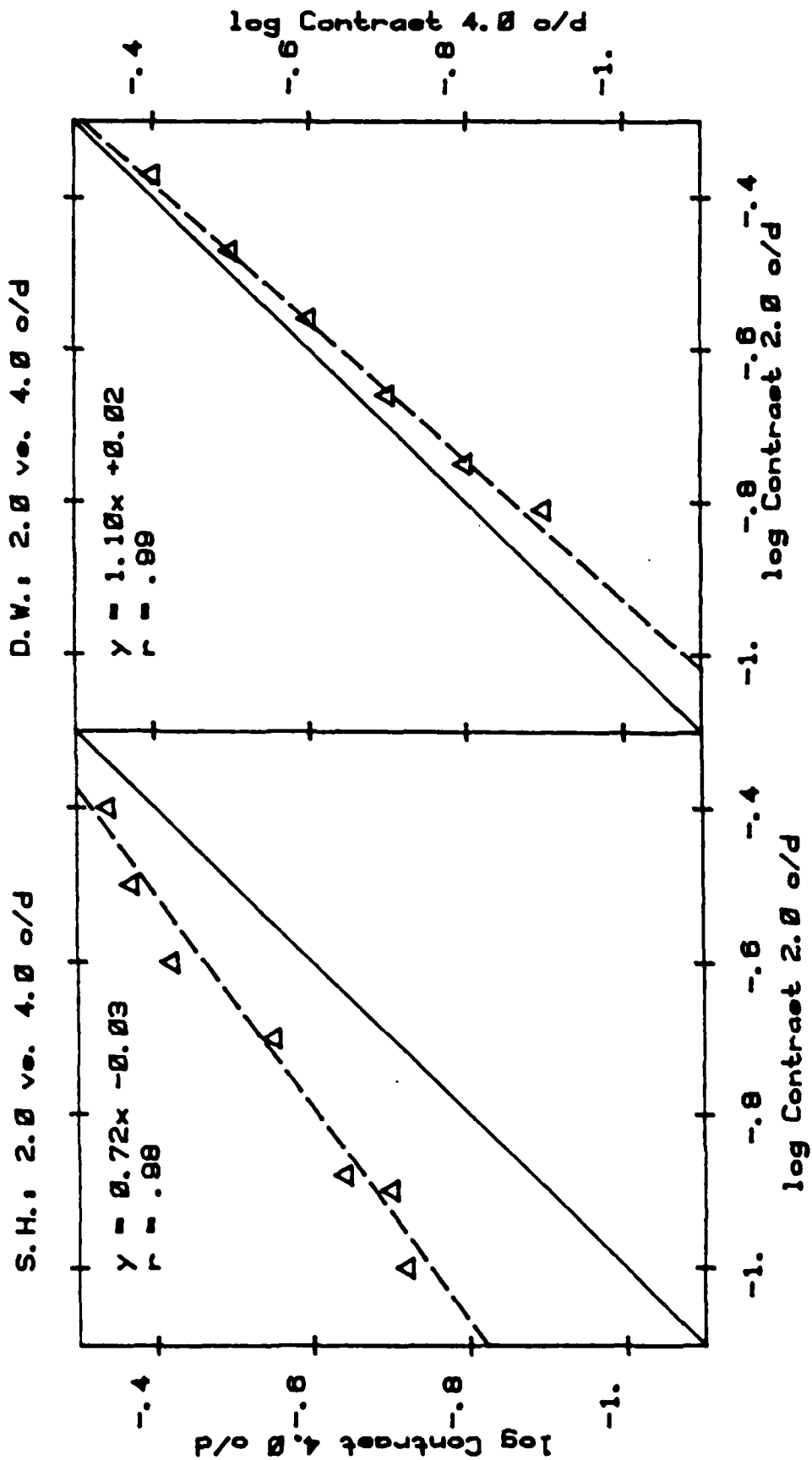


Figure 8F: contrast matching data for subjects S.H. & D.W.: 2.0 versus 4.0 o/d DOGs



Fig. 10 % Seen in motion, Subject S.H.  
DOG pairs by center frequency

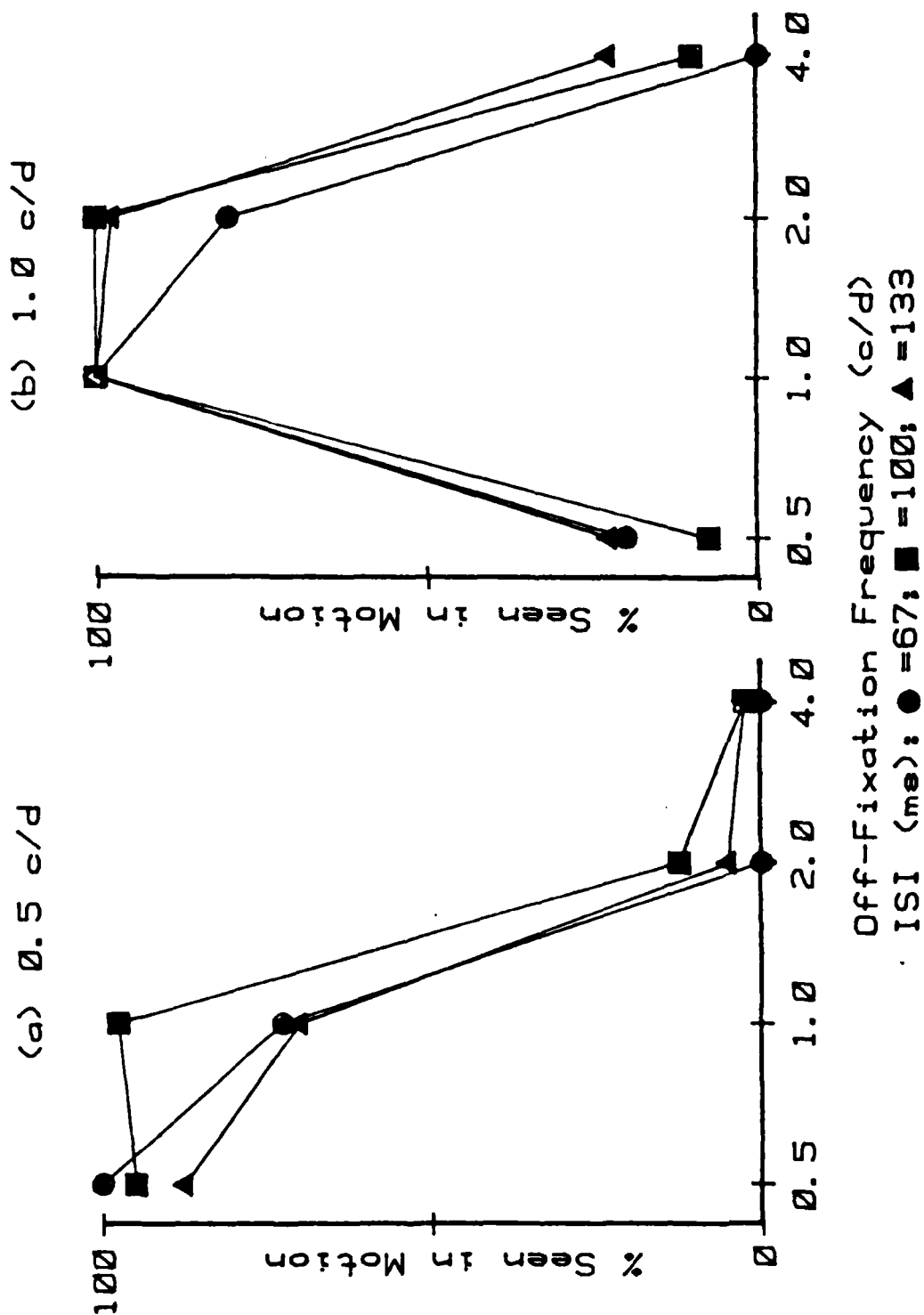


Fig. 10 % Seen in motion, Subject S.H.  
DOG pairs by center frequency

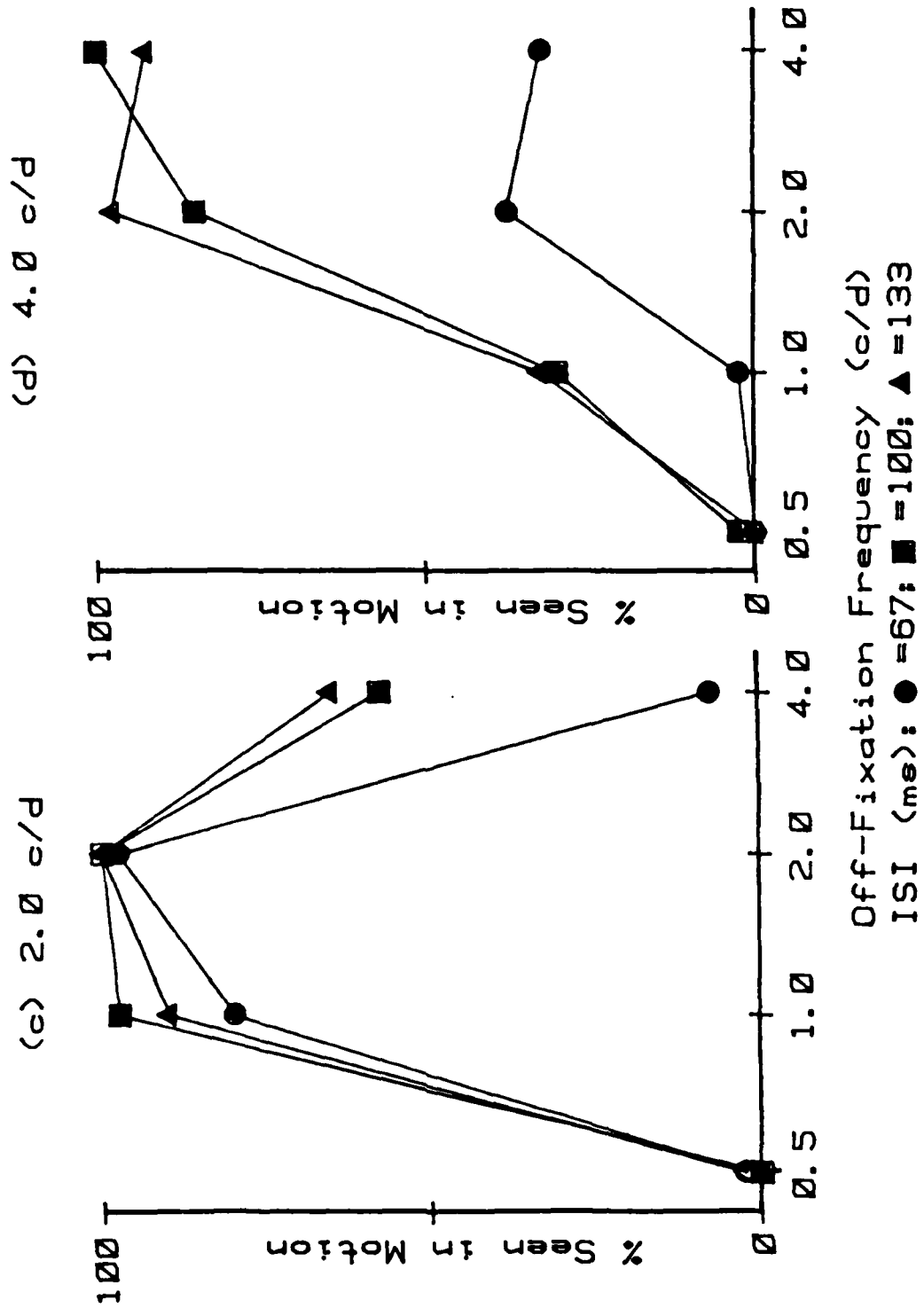


Fig. 11 % Seen in motion, Subject T.B.  
 DDC pairs by center frequency

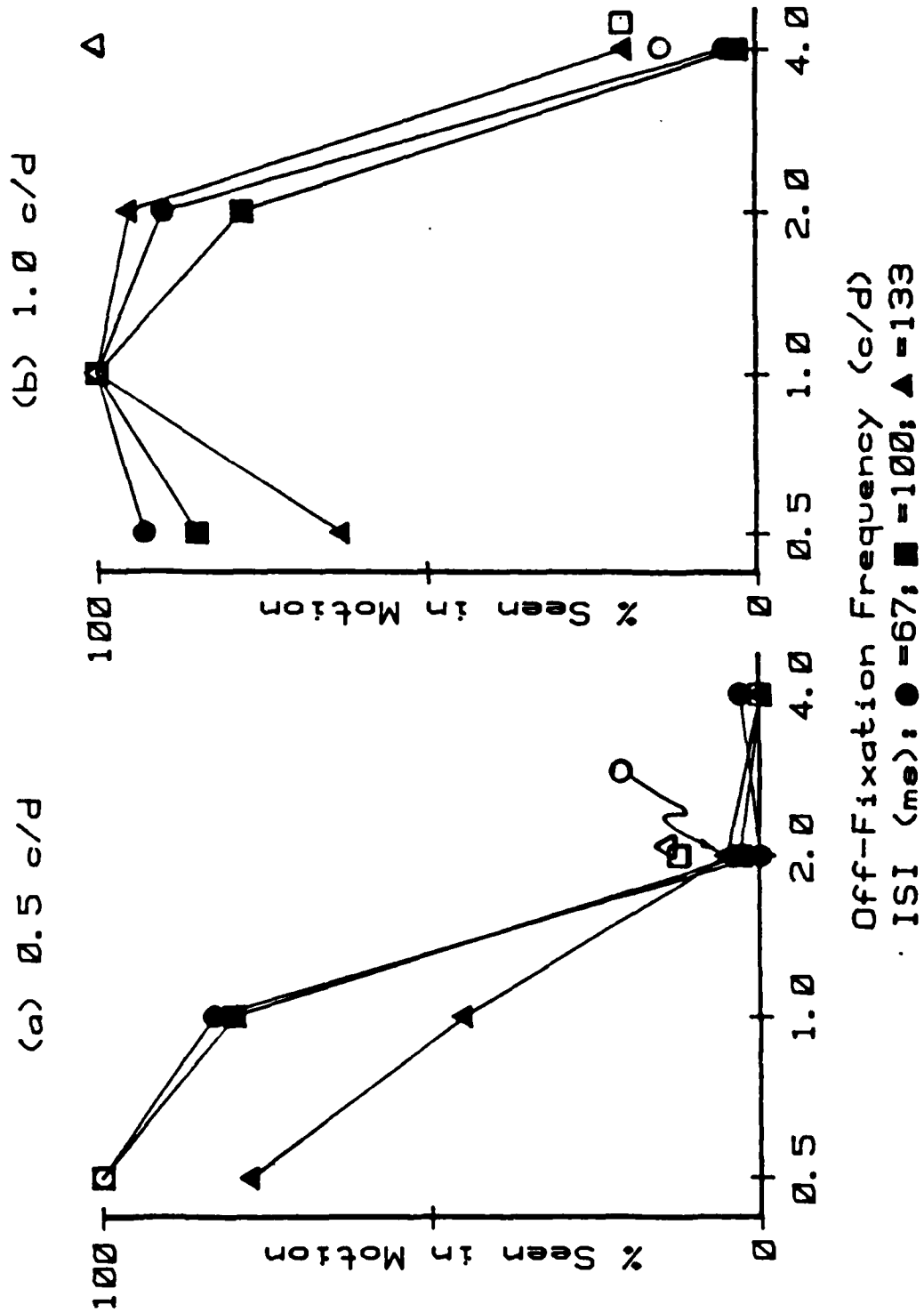


Fig. 11 % Seen in motion, Subject T.B.  
DOG pairs by center frequency

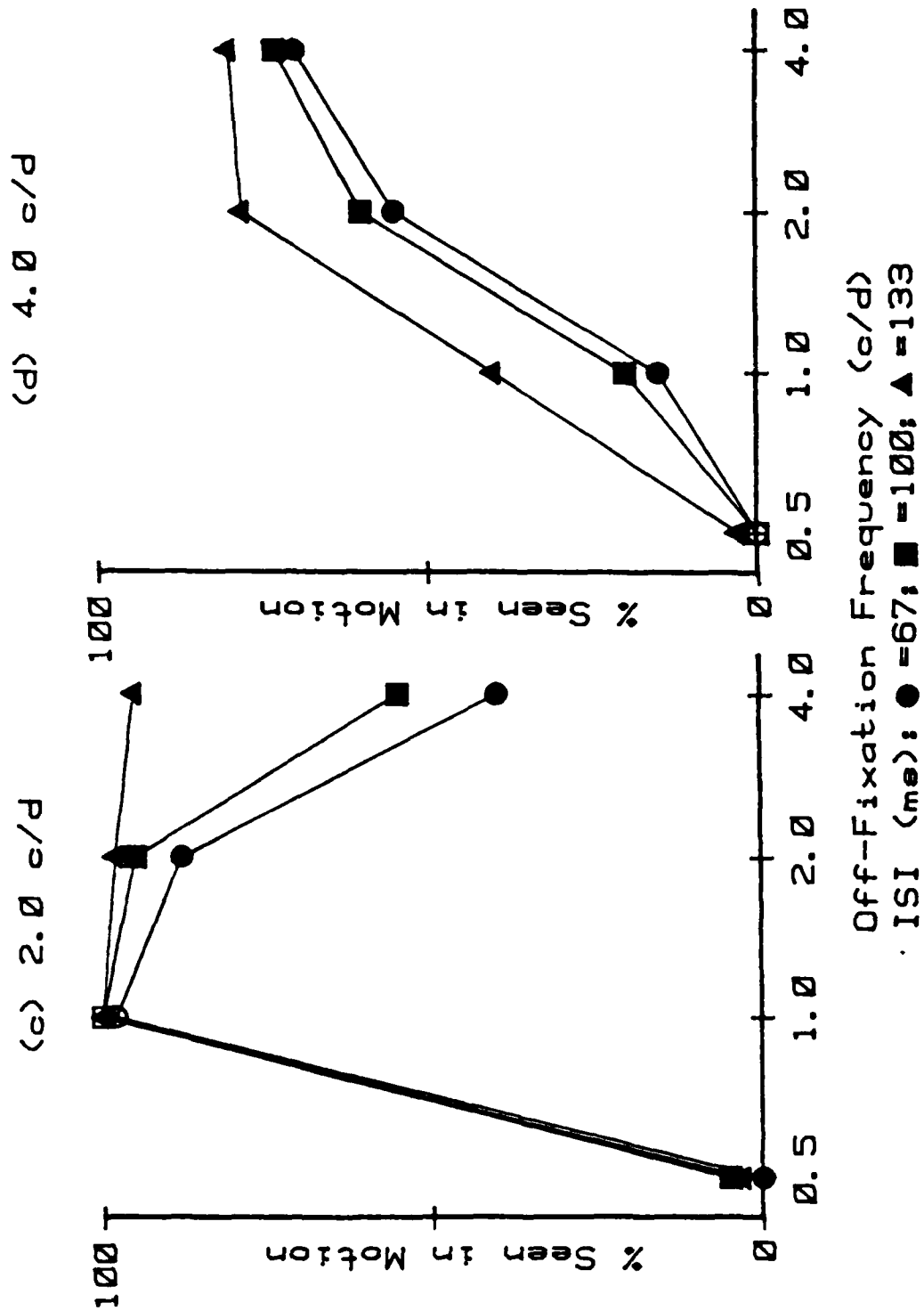


Fig. 12 % Seen in motion, Subject S.T.  
 DDC pairs by center frequency

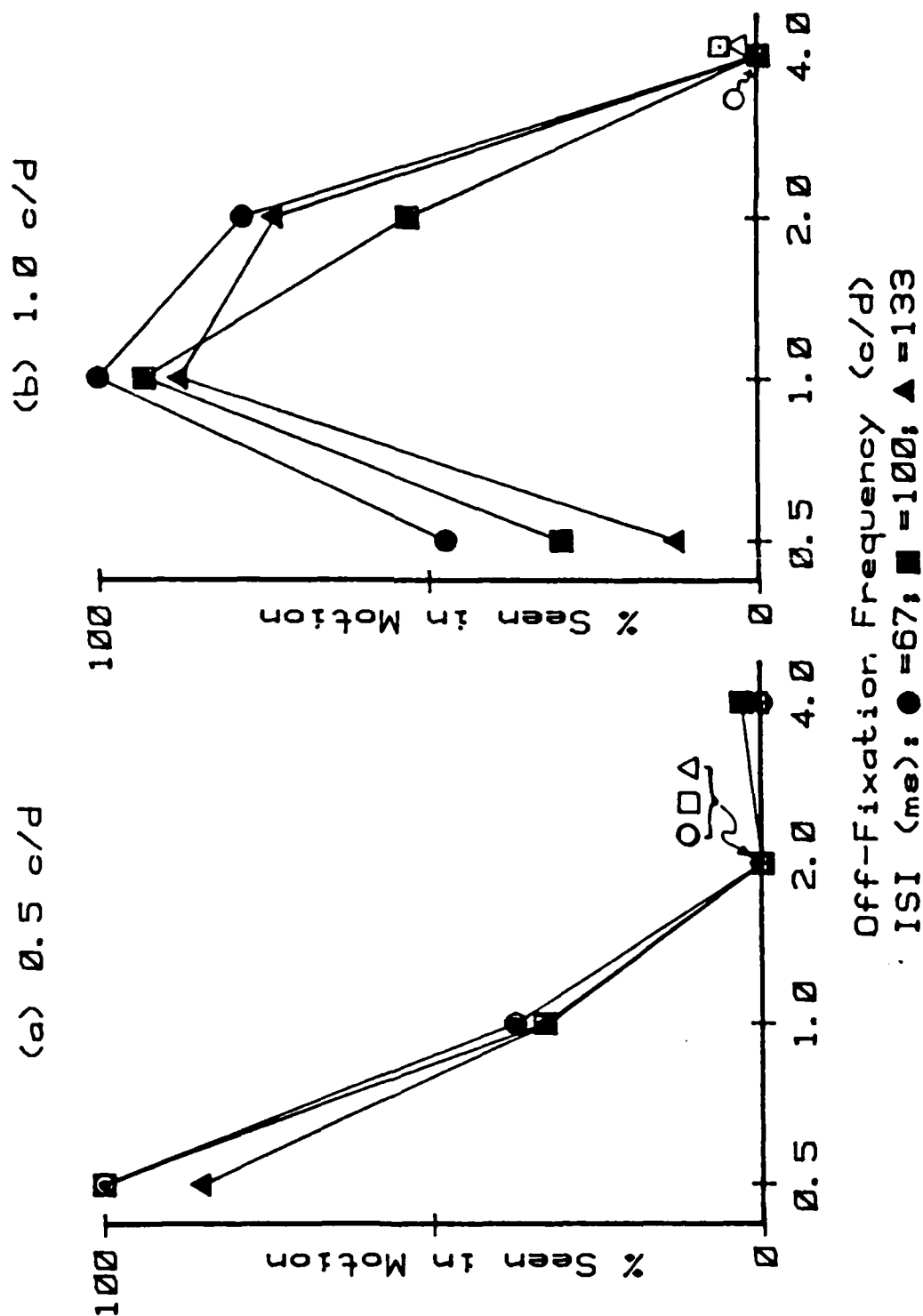


Fig. 12 % Seen in motion, Subject S.T.  
DOG pairs by center frequency

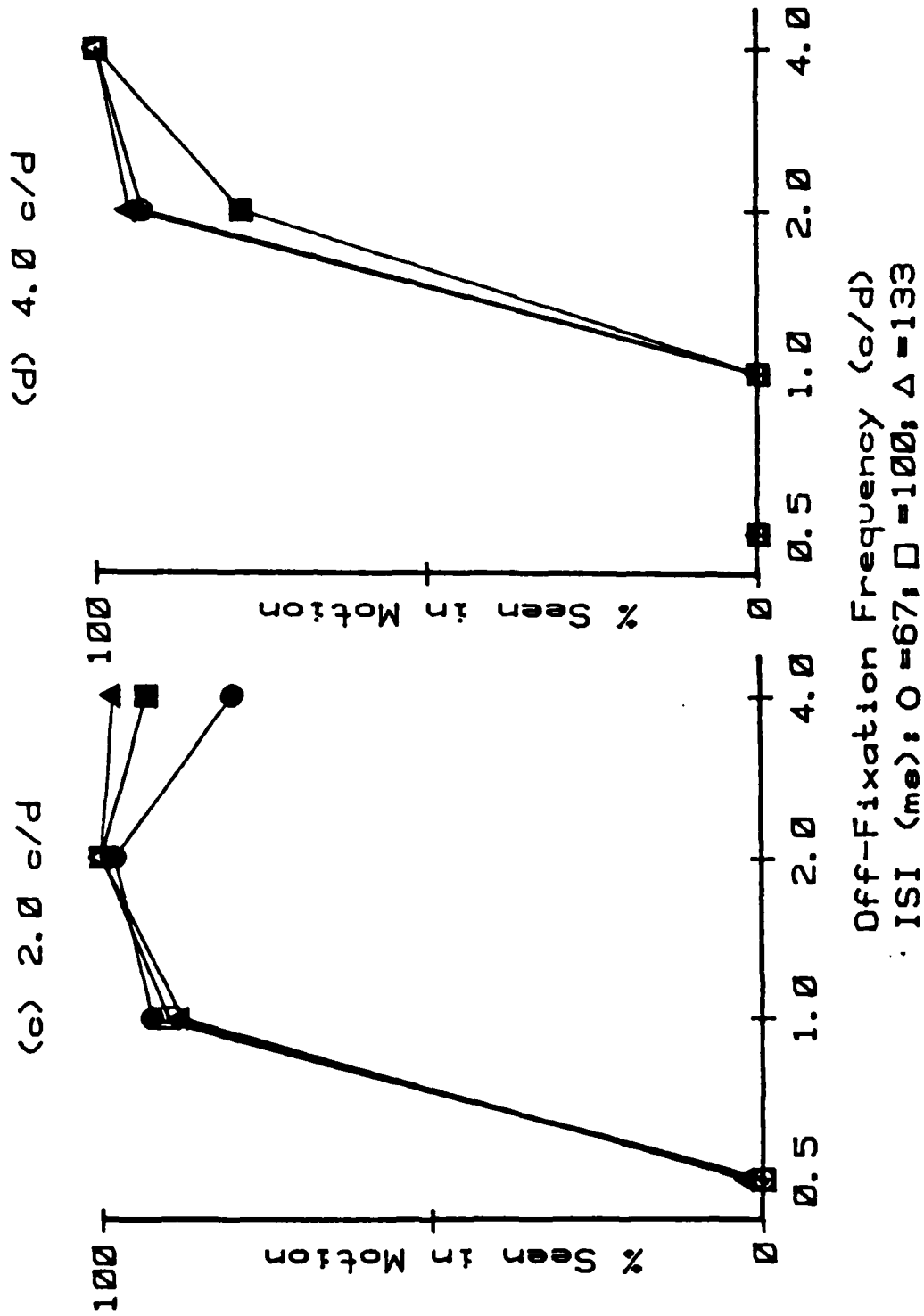


Fig. 13 % Seen in motion, Subject A.S.  
DOG pairs by center frequency

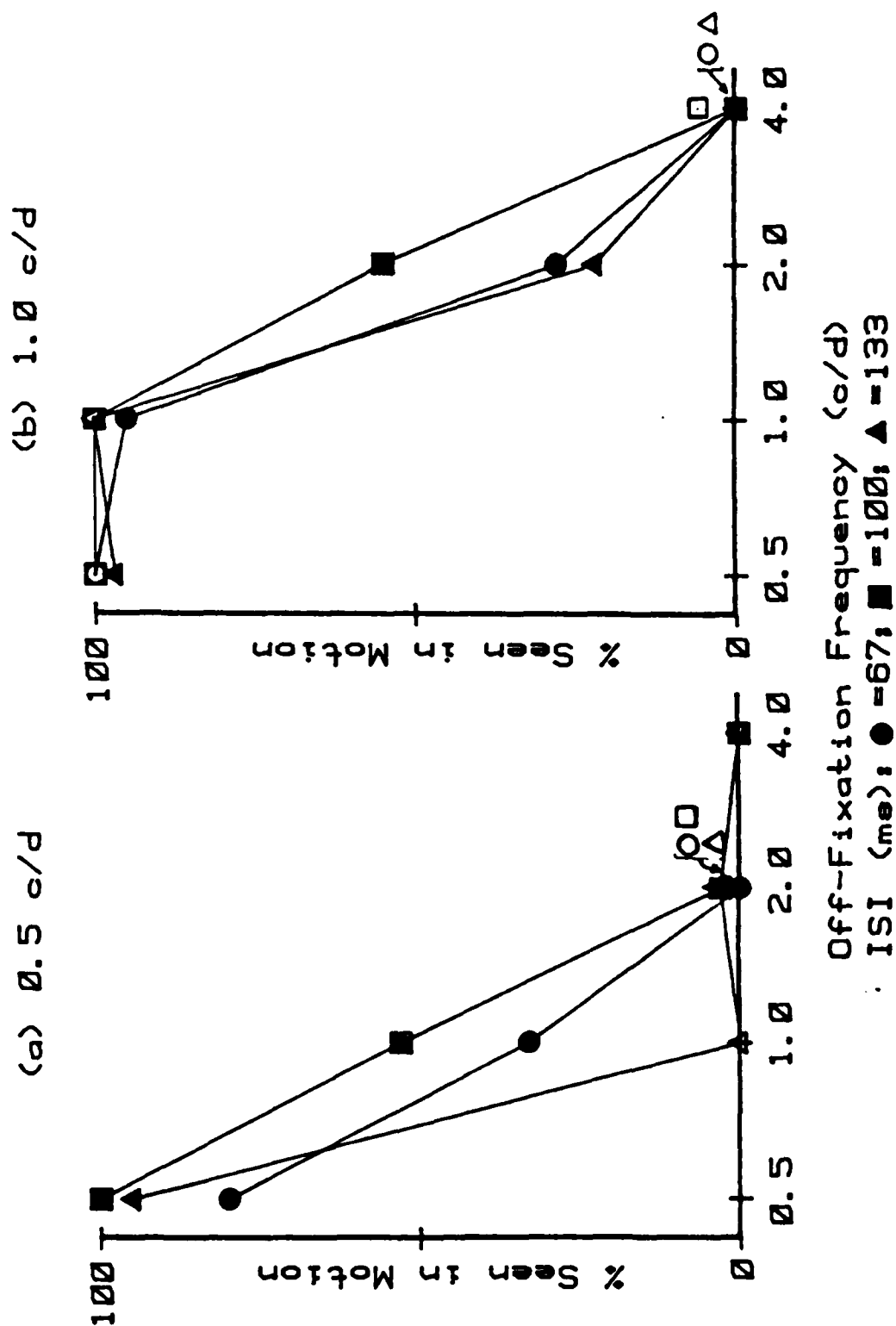


Fig. 13 % Seen in motion, Subject A.S.  
DOG pairs by center frequency

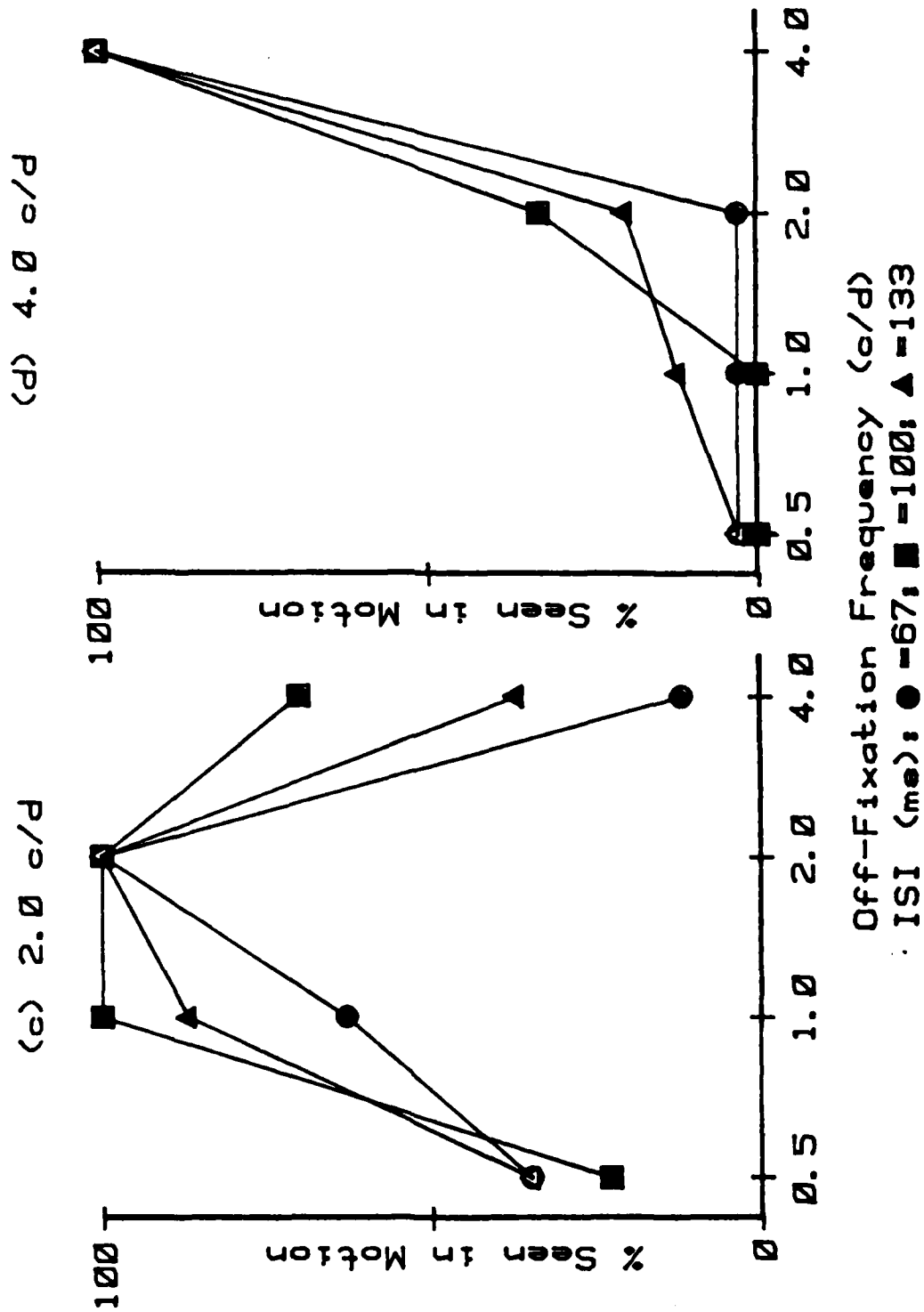
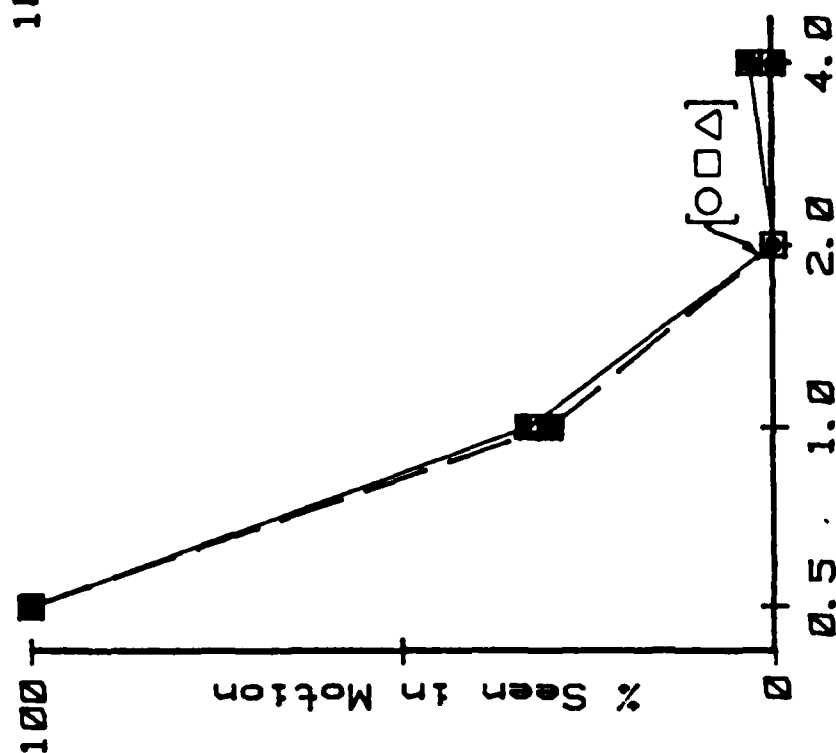


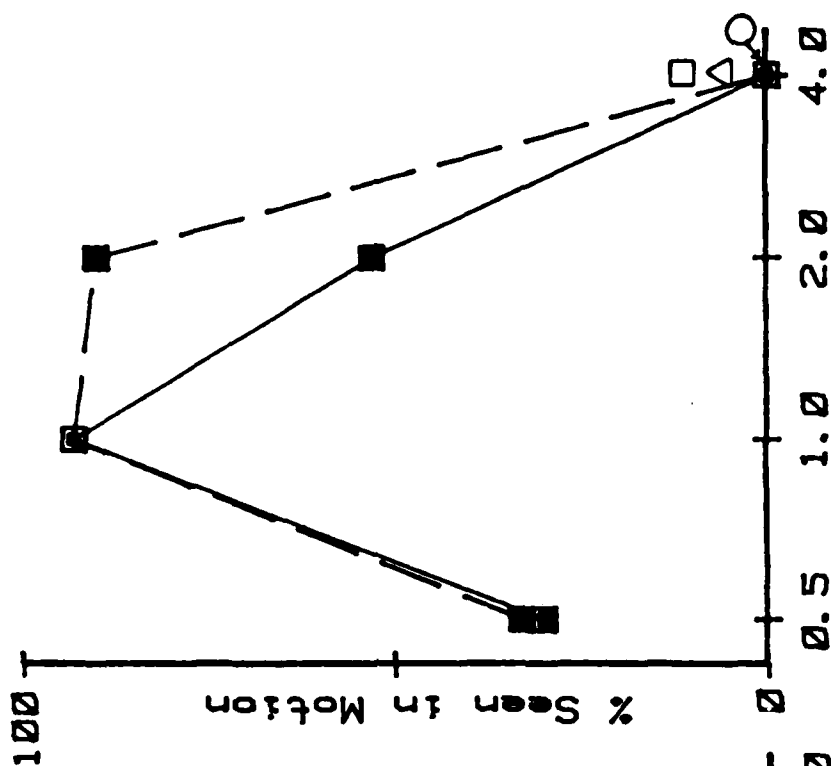


Fig. 14 % Seen in motion, Subject S.T.  
Pairs by center & flank freq.

(a) 0.5 o/d

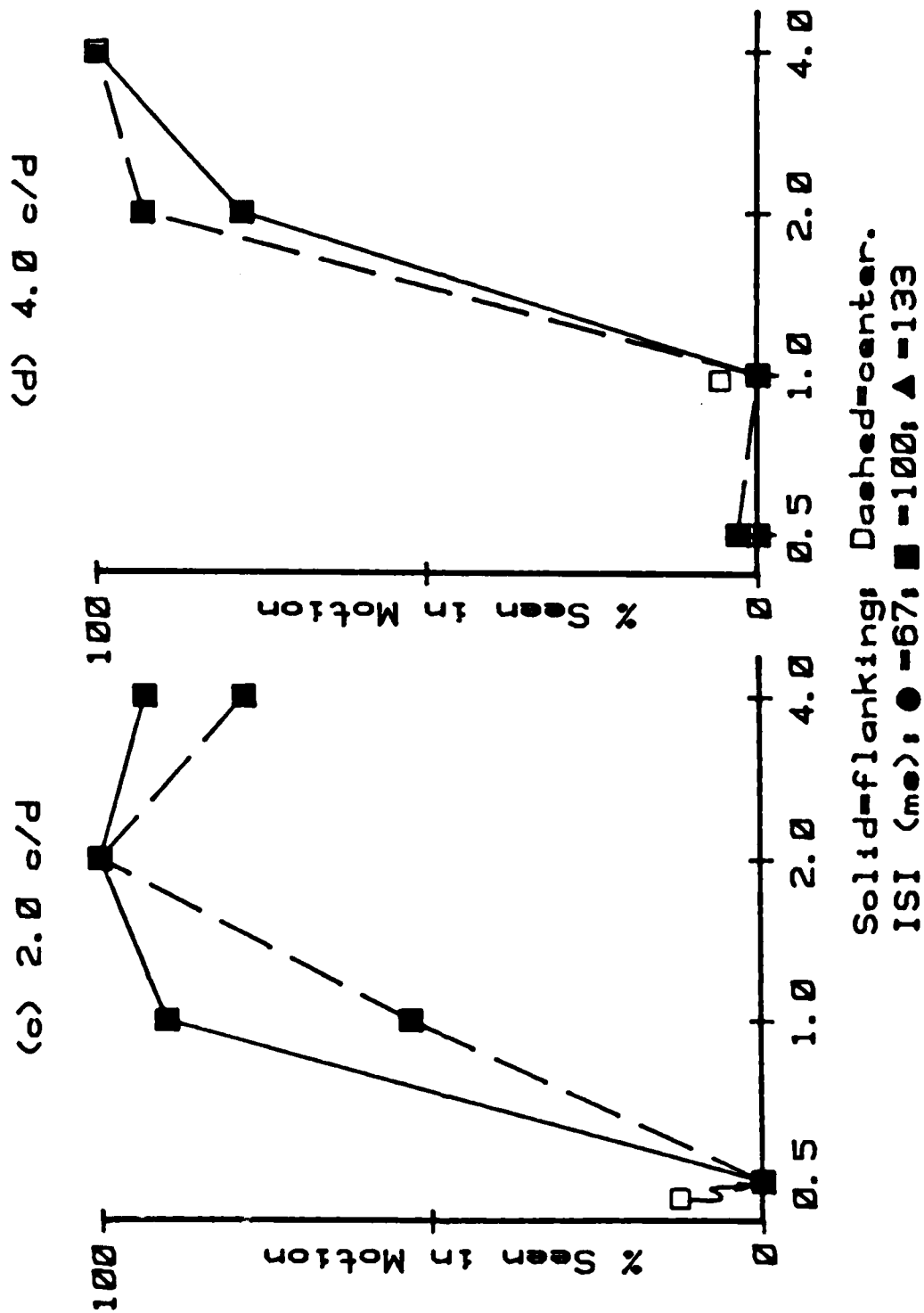


(b) 1.0 o/d



Solid=flanking; Dashed=center.  
ISI (ms): ● =67; ■ =100; ▲ =133

Fig. 14 % Seen in motion, Subject S.T.  
Pairs by center & flank freq.



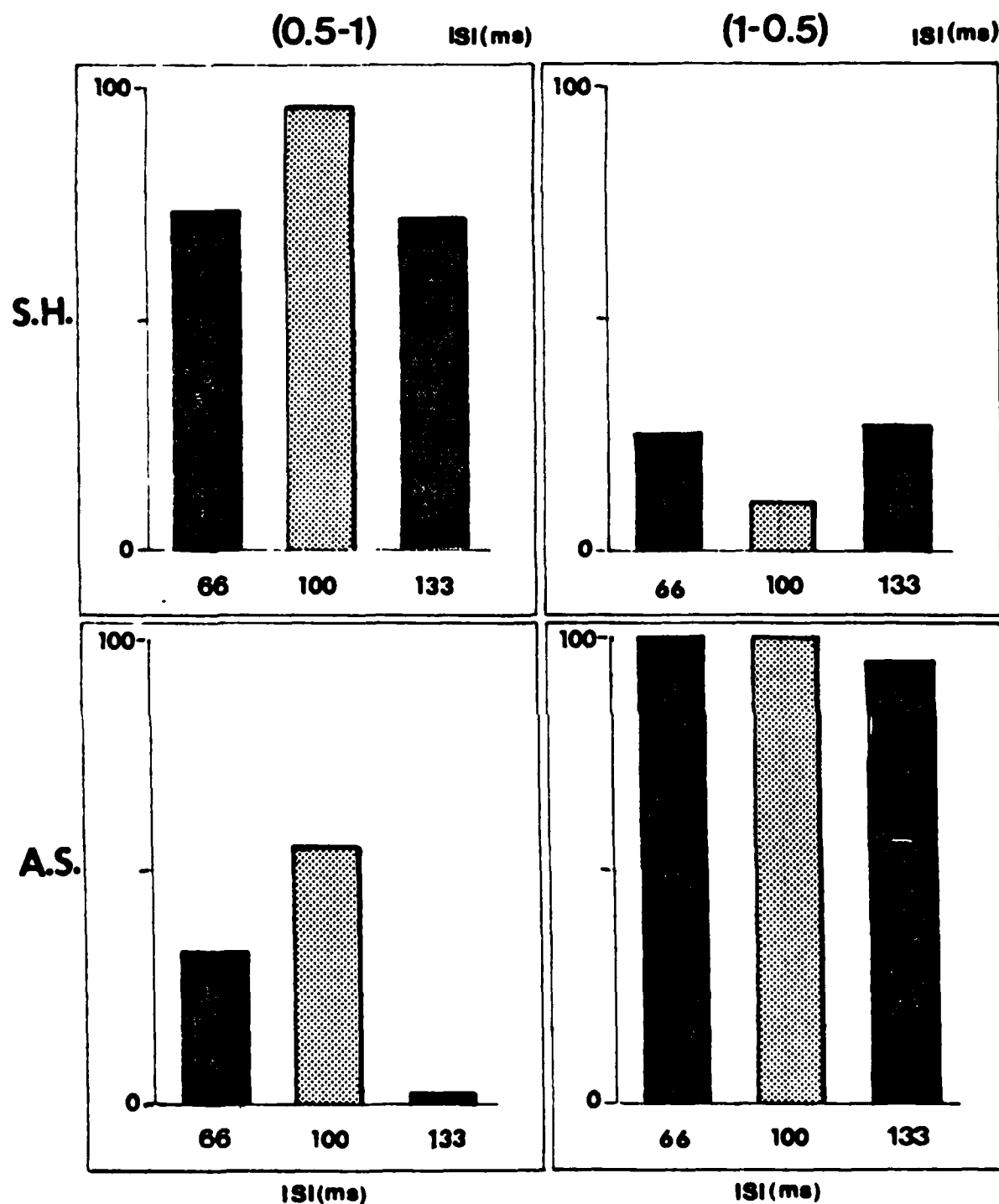


FIGURE 15 ANOMALOUS RESULTS IN EXPERIMENT 1 FOR SUBJECTS S.H. & A.S. (TOP AND BOTTOM ROWS). PAIRS ( $\frac{1}{2}$ -1), LEFT COLUMN, & ( $1-\frac{1}{2}$ ) RIGHT COLUMN, SHOW CONSIDERABLE DIFFERENCE.

Fig. 16 % Seen in motion, Composite.  
 DDC pairs by either frequency

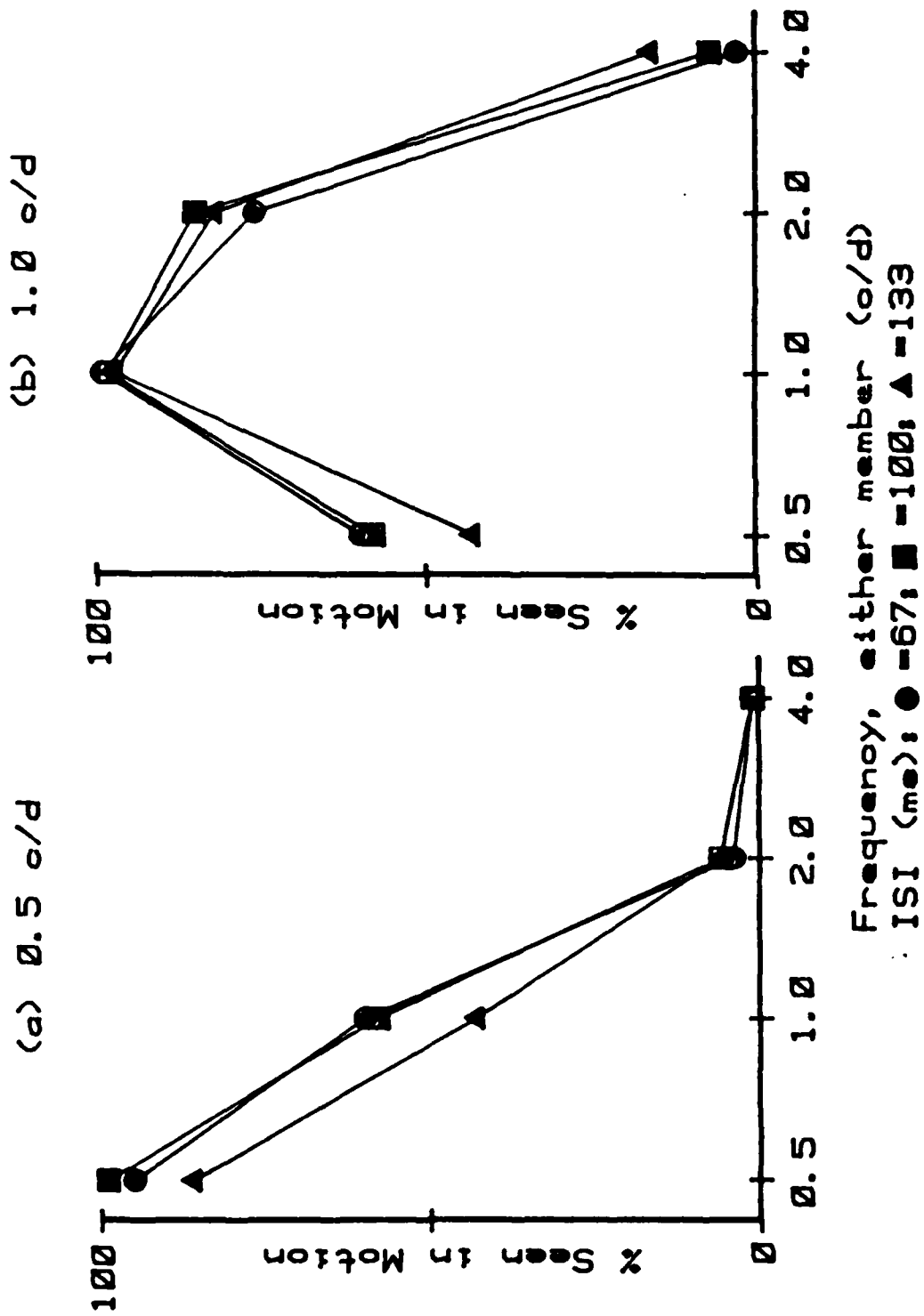
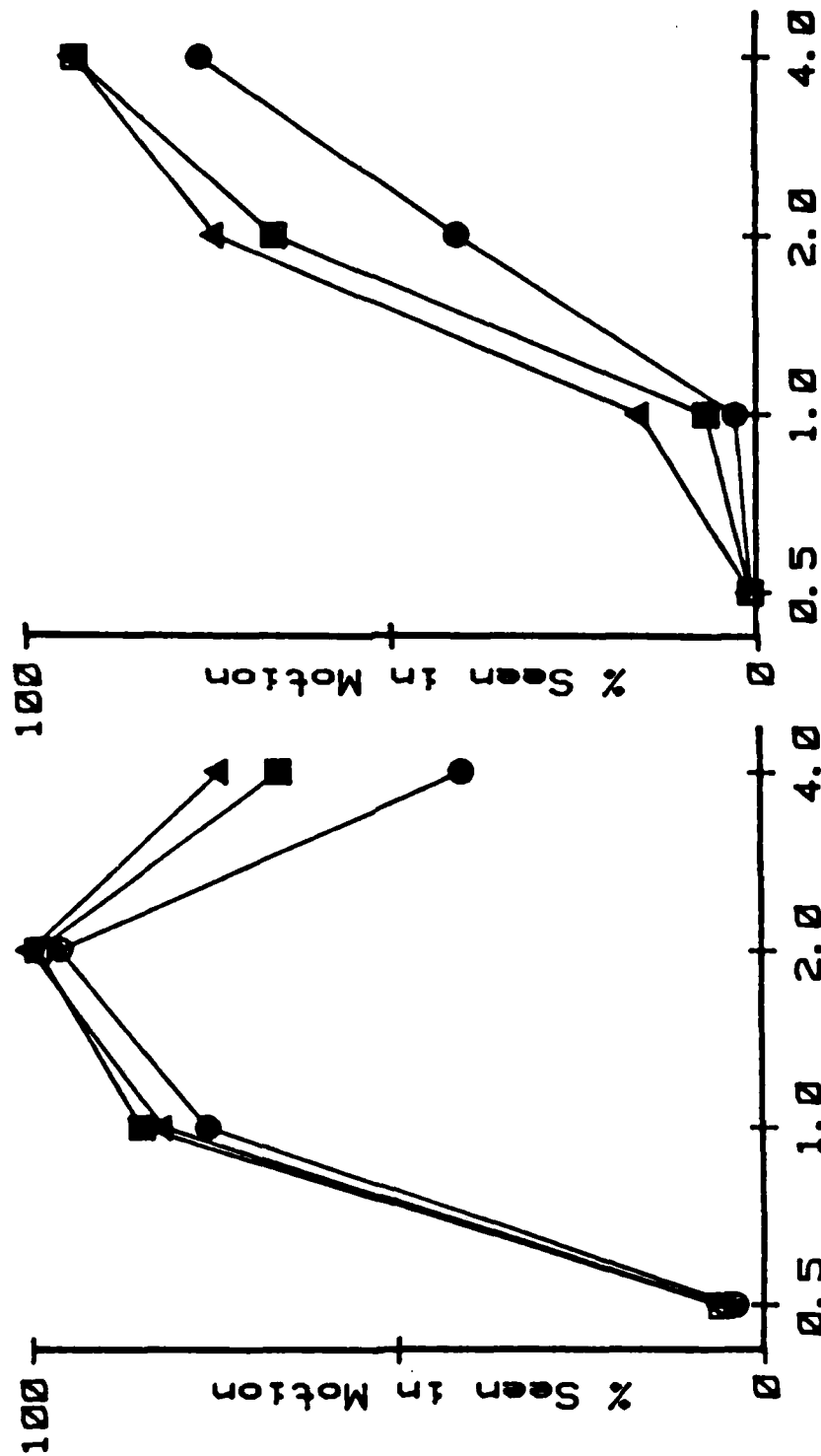


Fig. 16 % Seen in motion, Composite.  
DOG pairs by either frequency

(d) 4.0 o/d



Frequency, either member (o/d)  
ISI (me); ● -67; ■ -100; ▲ -133

DIFFERENCE IN PERCENT SEEN IN MOTION AT 67MS ISI AND  
AT 133MS ISI.

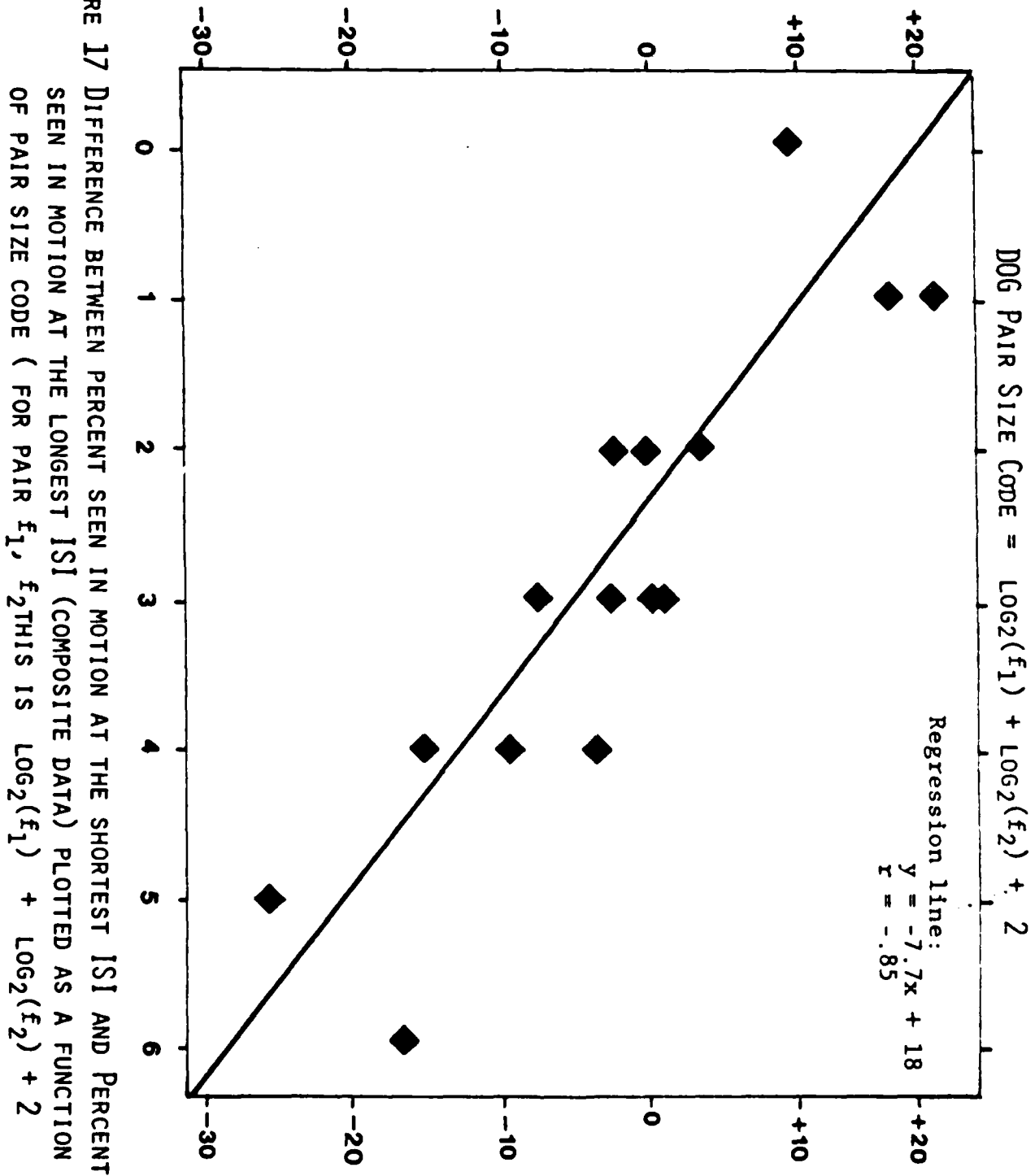


FIGURE 17 DIFFERENCE BETWEEN PERCENT SEEN IN MOTION AT THE SHORTEST ISI AND PERCENT SEEN IN MOTION AT THE LONGEST ISI (COMPOSITE DATA) PLOTTED AS A FUNCTION OF PAIR SIZE CODE ( FOR PAIR  $f_1$ ,  $f_2$  THIS IS  $\log_2(f_1) + \log_2(f_2) + 2$

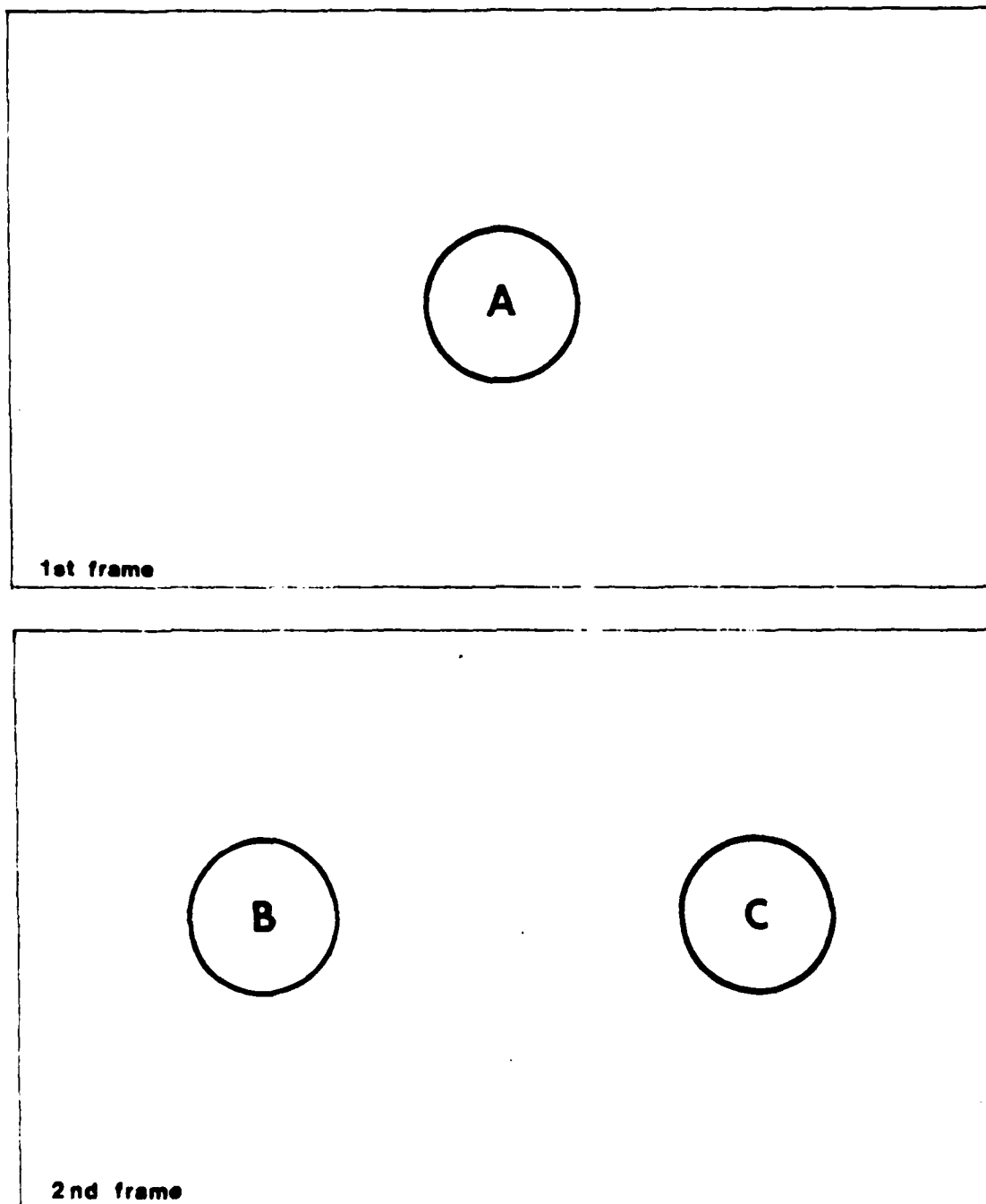


FIGURE 18 THE SPLITTING CONFIGURATION. WHEN THE TWO FRAMES ARE ALTERNATED, OBJECT A MAY APPEAR TO MOVE TO THE LEFT SIDE (B), THE RIGHT SIDE (C) OR BOTH SIDES SIMULTANEOUSLY, I.E. SPLIT.

# S.H. [1/2 c/d Centers]

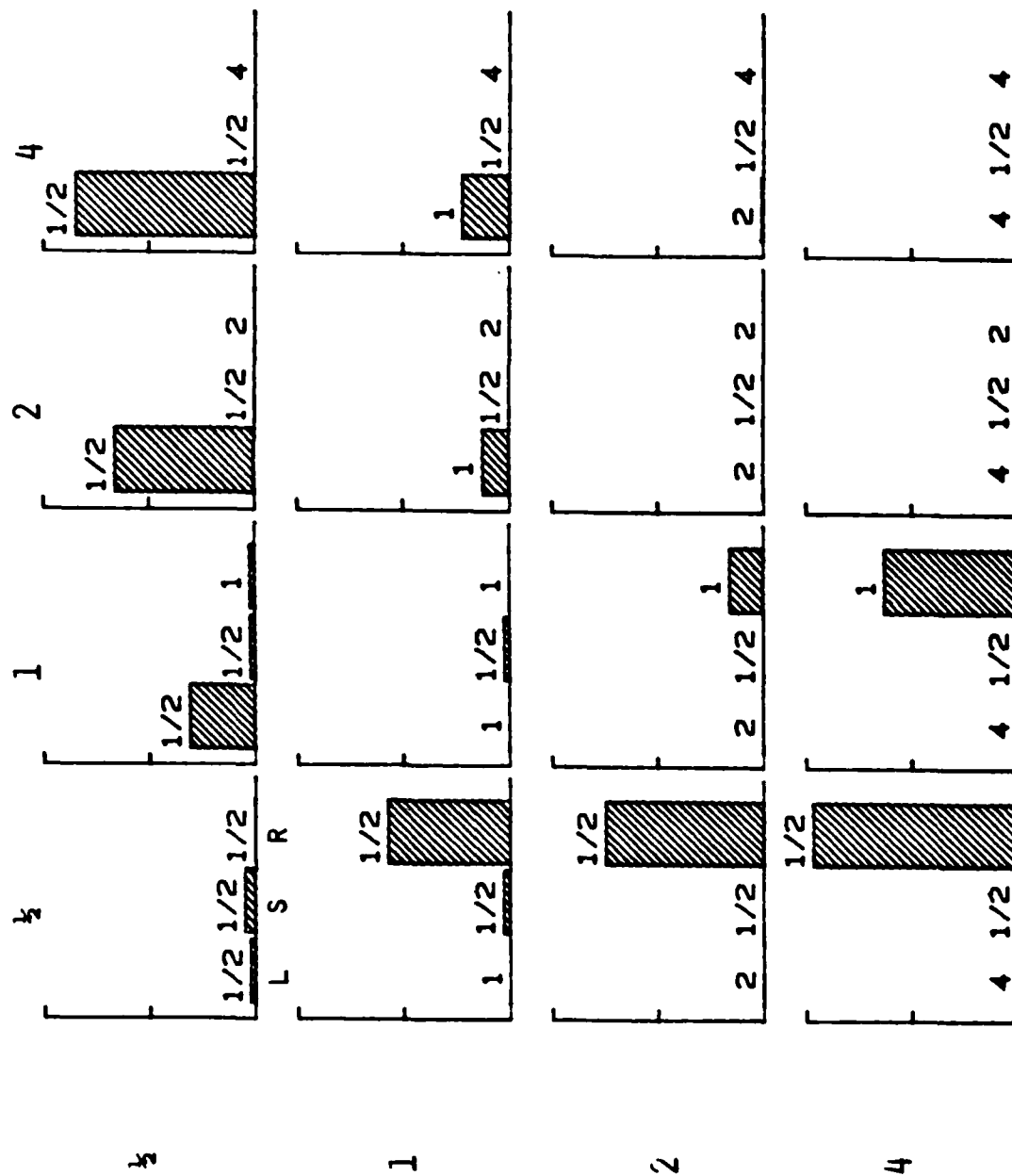


FIGURE 19A (SEE TEXT)



# A.S. [1/2 c/d Centers]

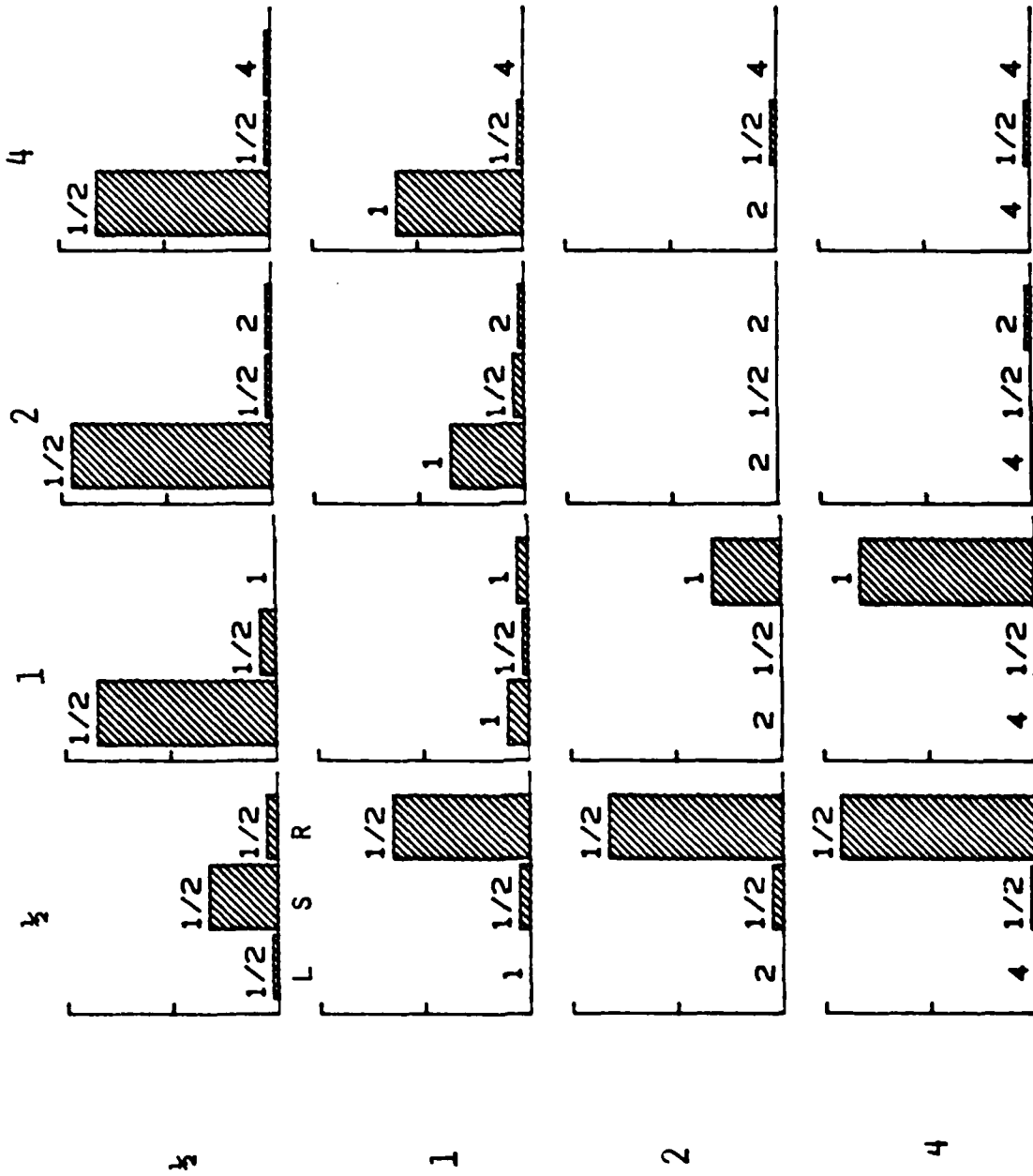


FIGURE 19B (SEE TEXT)

# S.T. [1/2 c/d Centers]

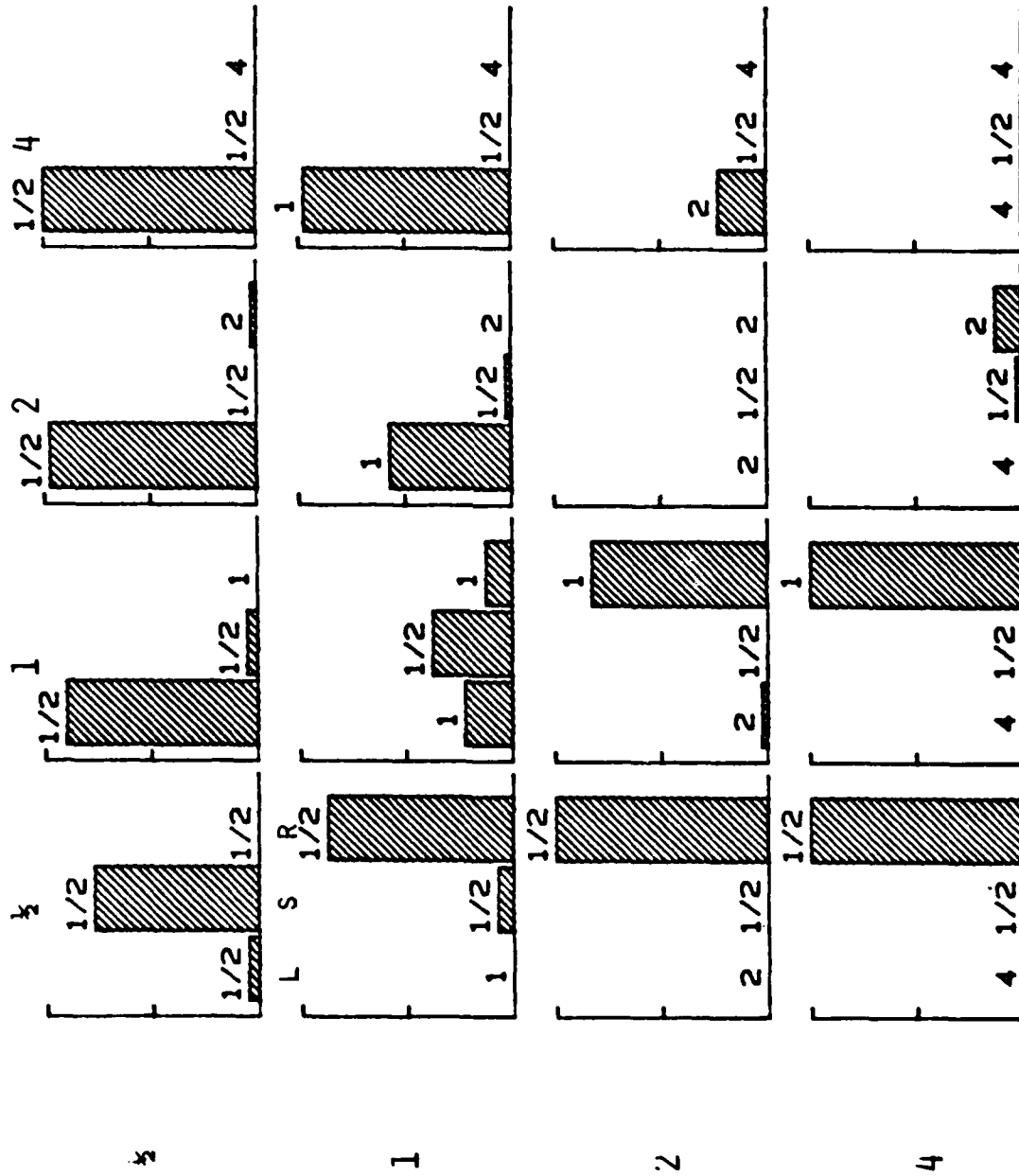


FIGURE 19c (SEE TEXT)

# N.L. [1/2 c/d Centers]

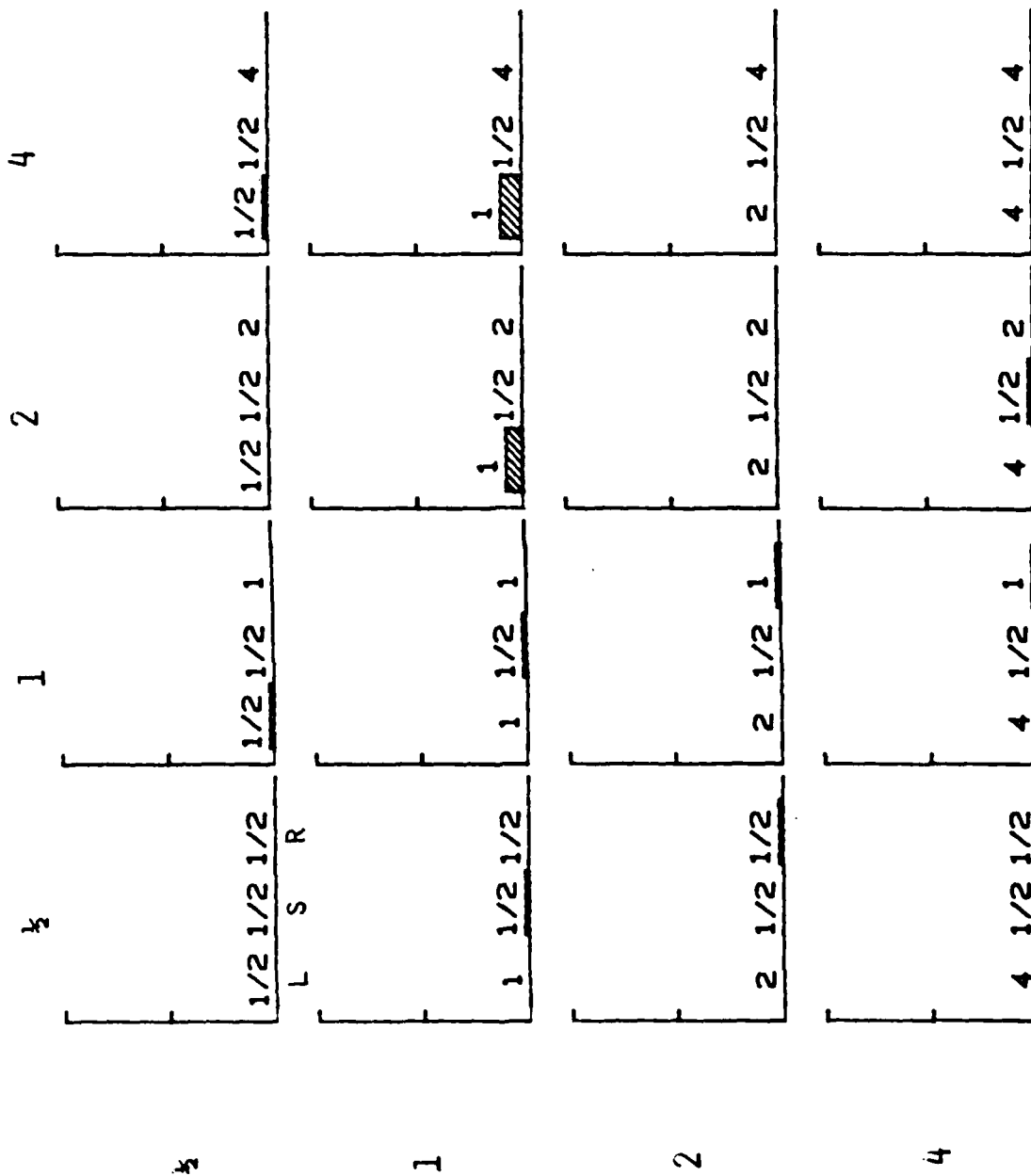


FIGURE 19d (SEE TEXT)

# S.H. [1 c/d Centers]

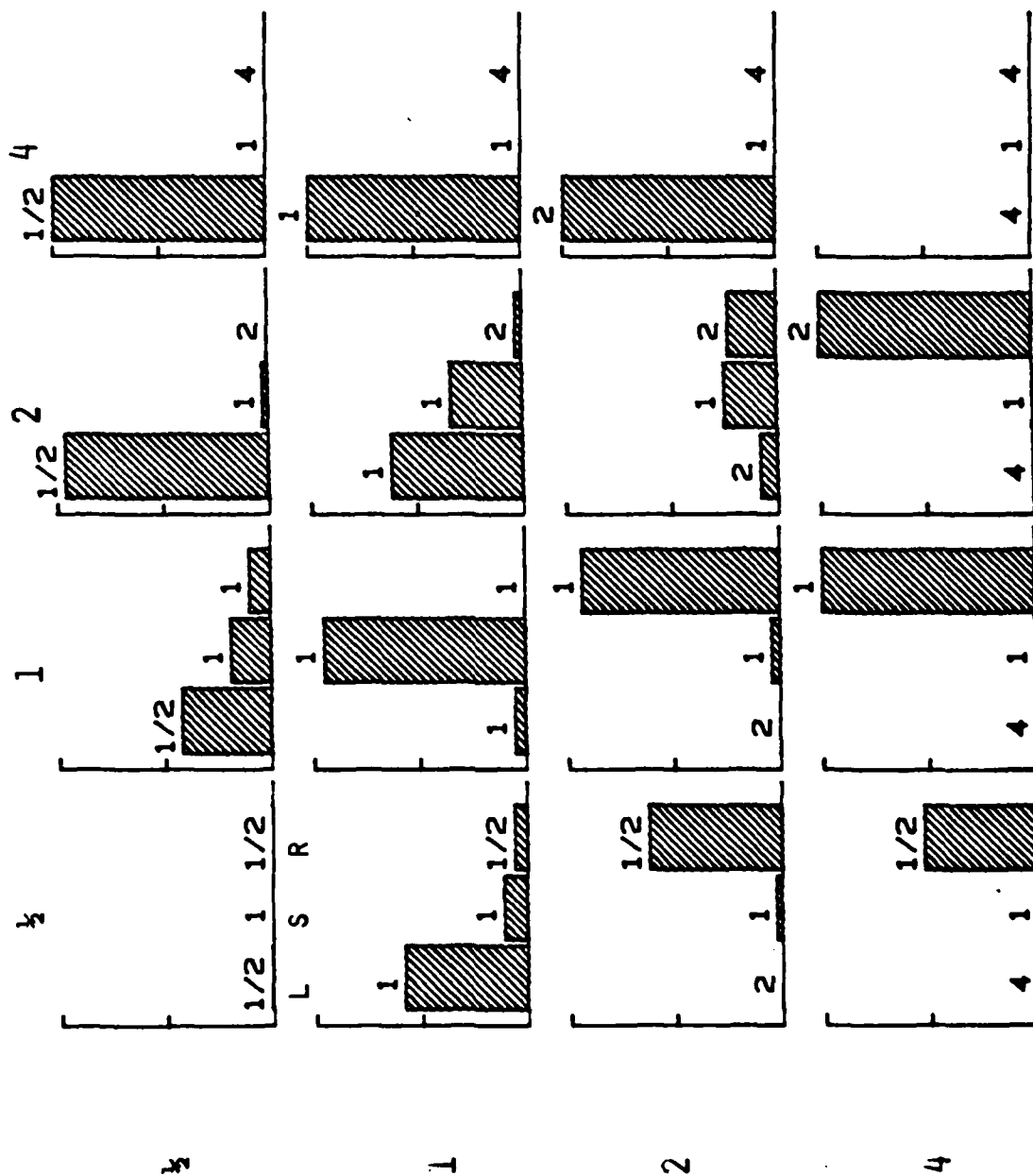


FIGURE 20a (SEE TEXT)

# A.S. [1 c/d Centers]

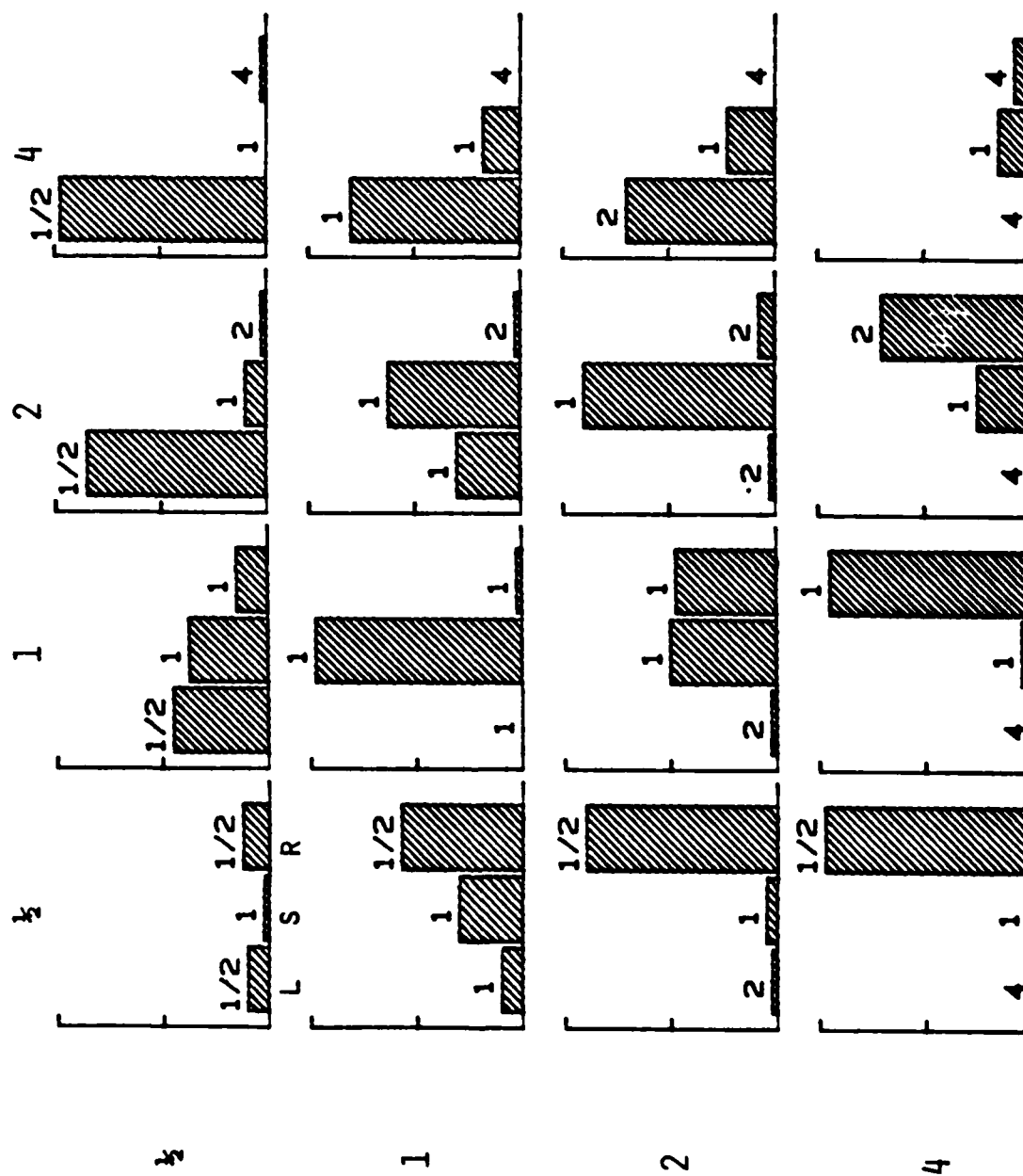


FIGURE 20B (SEE TEXT)

# S.T. [1 c/d Centers]

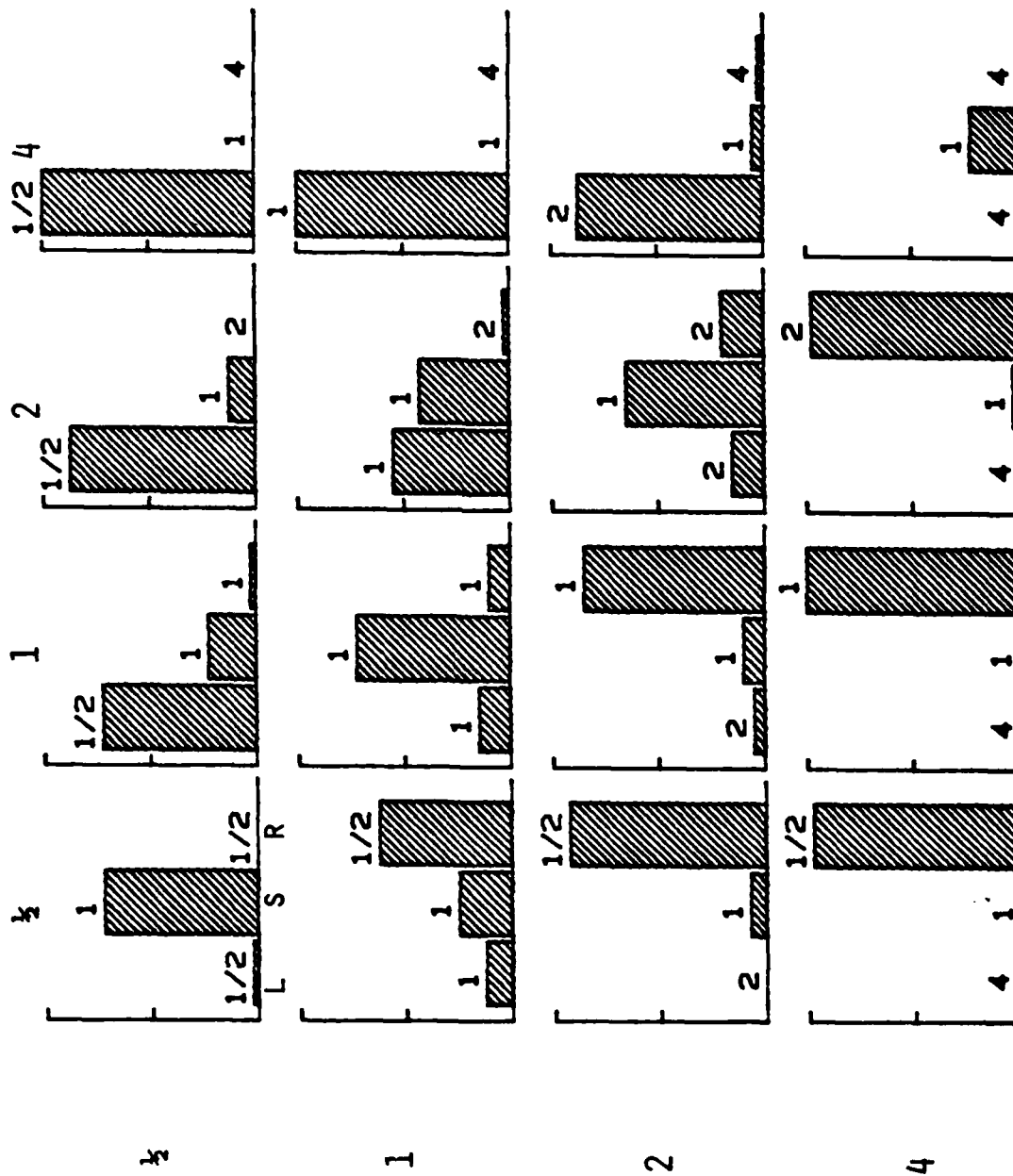


FIGURE 20c (SEE TEXT)

# N.L. [1 c/d Centers]

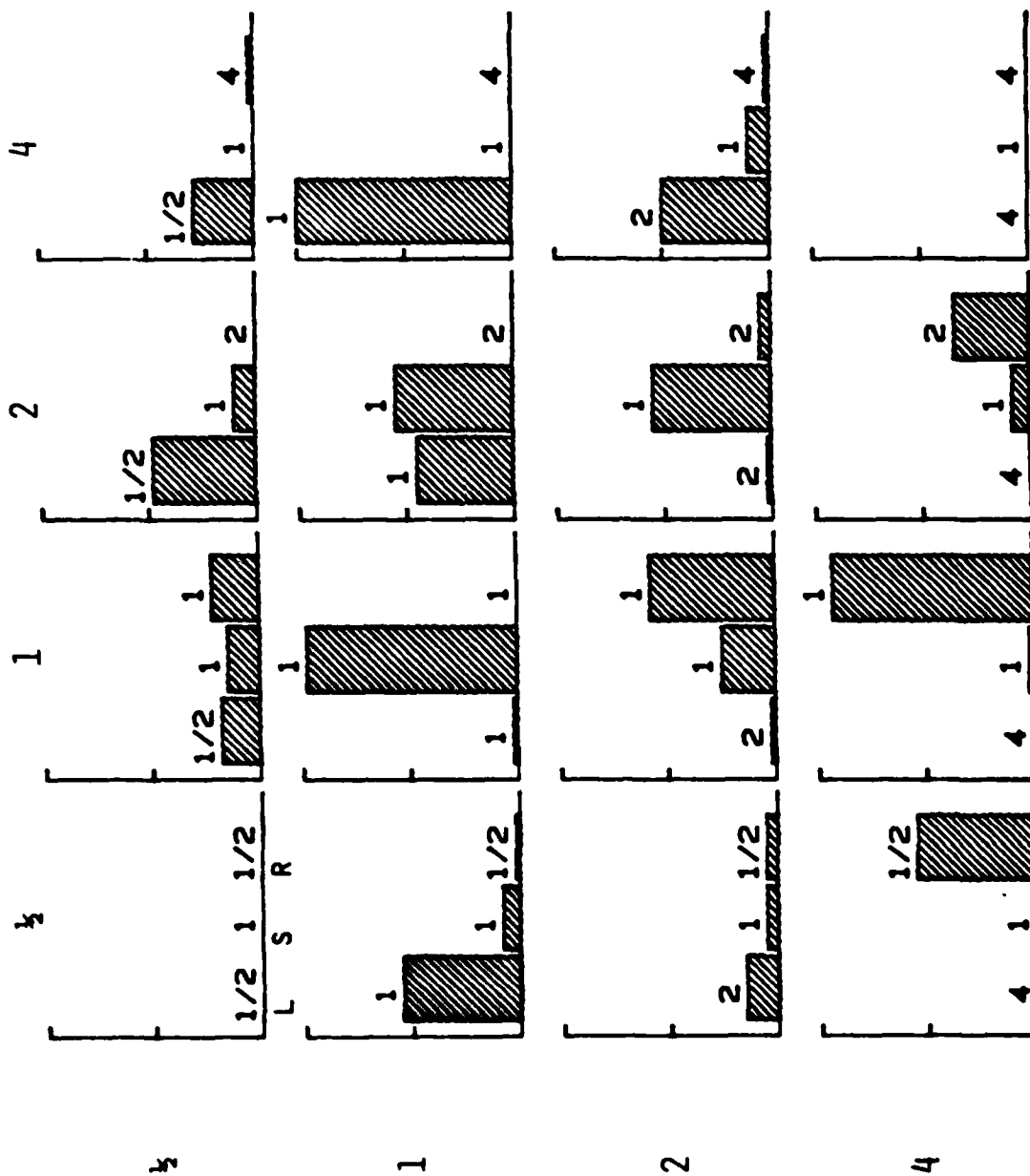


FIGURE 20D (SEE TEXT)

# S.H. [2 c/d Centers]

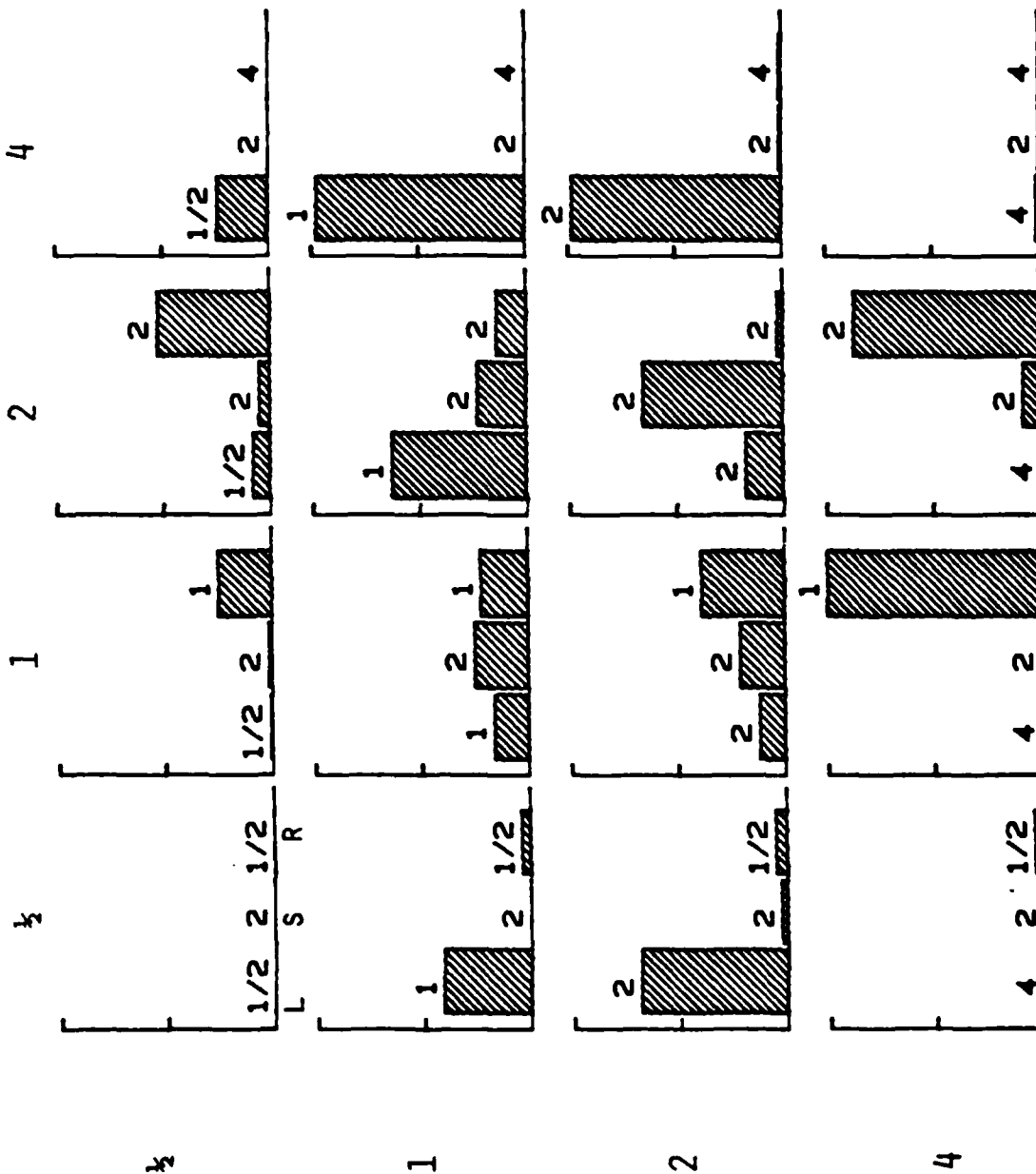


FIGURE 21A (SEE TEXT)



# A.S. [2 c/d Centers]

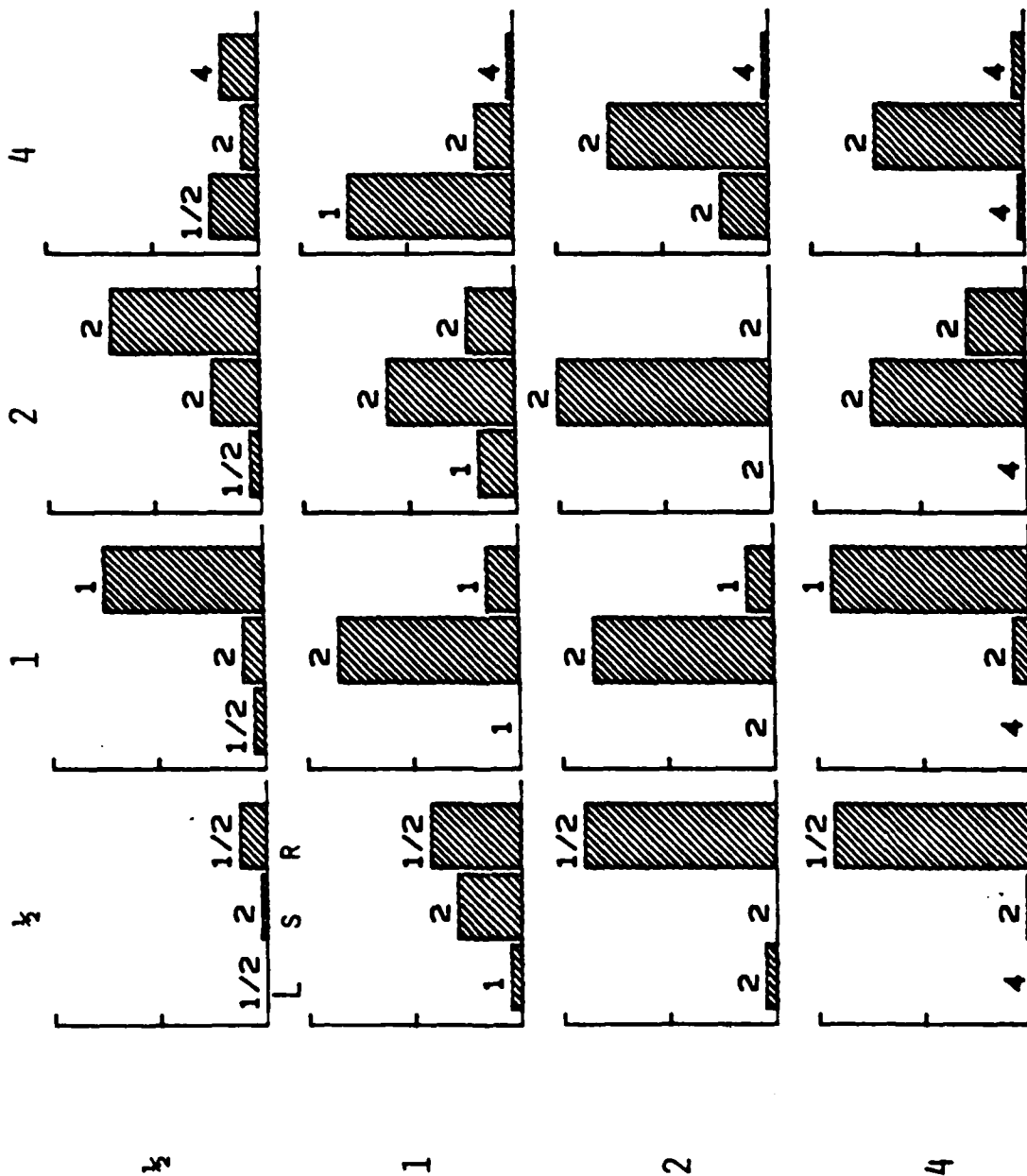


FIGURE 21b (SEE TEXT)

# S.T. [2 c/d Centers]

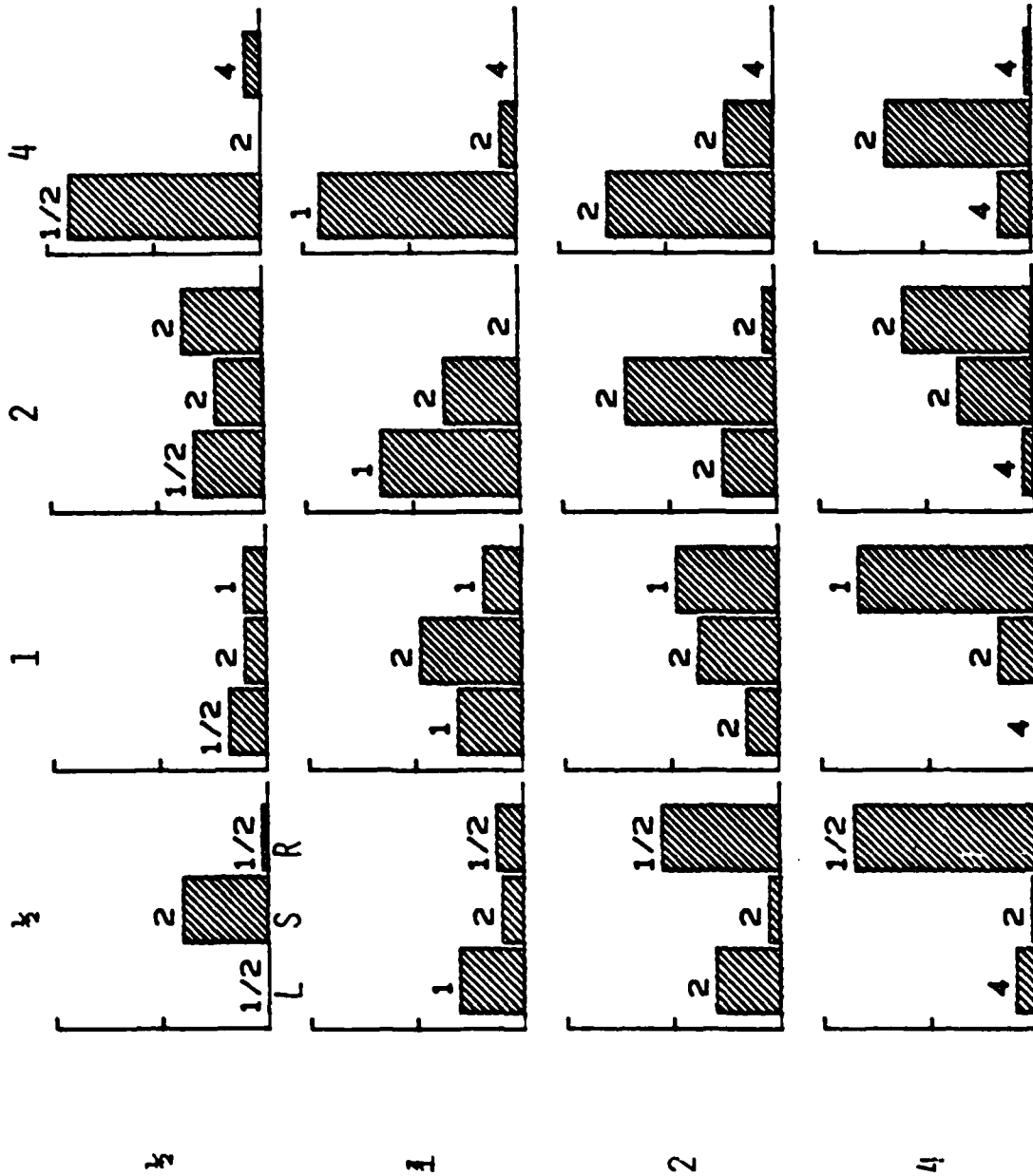


FIGURE 21c (SEE TEXT)

# N.L. [2 c/d Centers]

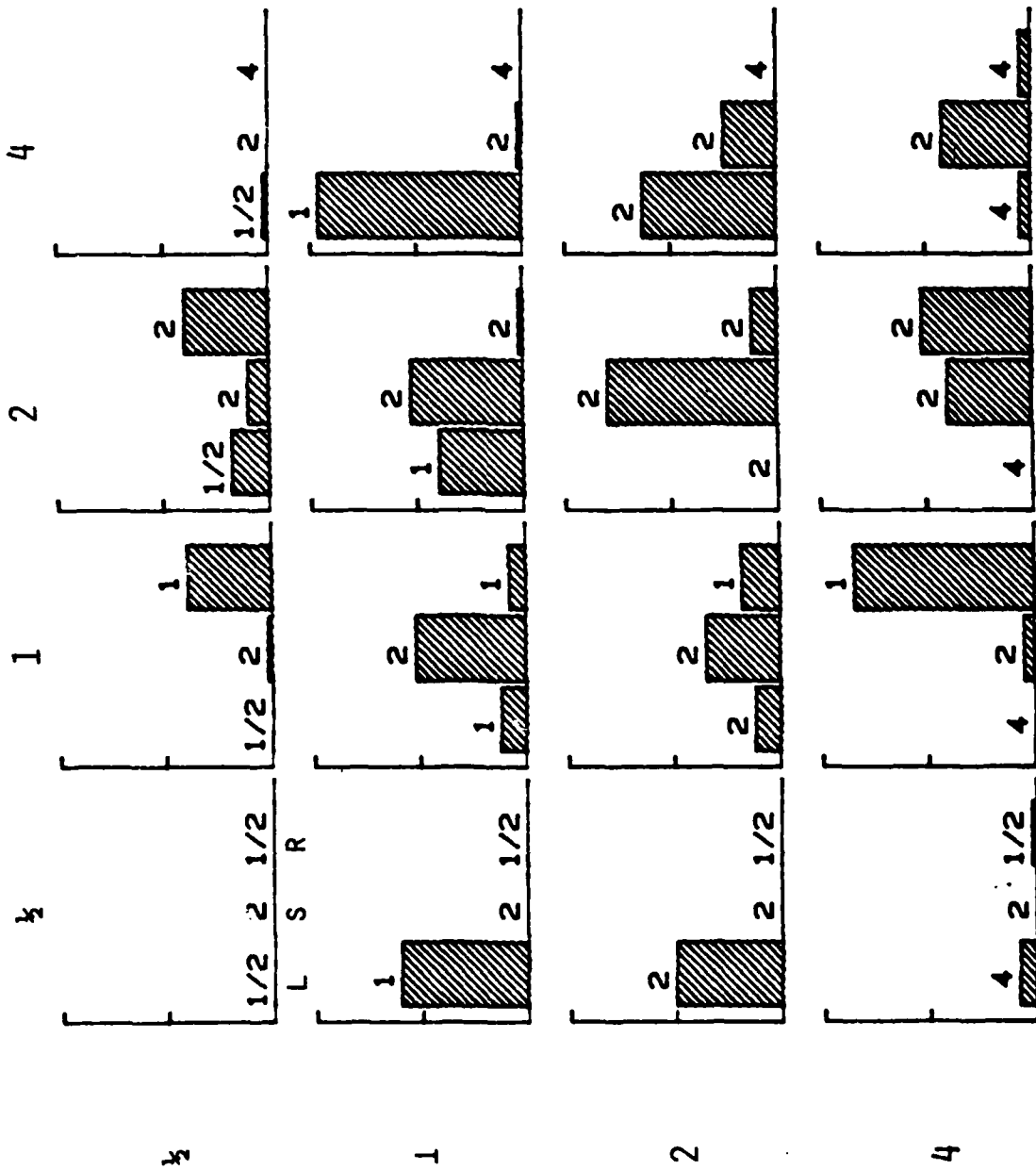


FIGURE 21d (SEE TEXT)

Figure 1 displays a 4x4 grid of bar charts. The columns are labeled L, S, R, and the rows are labeled  $k$ . The bars are shaded with diagonal lines. The values for each bar are indicated by numbers above or below them.

$k$	L	S	R	$k$
4	4	4	4	4
2	2	2	2	2
1	1	1	1	1
1/2	1/2	1/2	1/2	1/2

**FIGURE 22A (SEE TEXT)**

# A.S. [4 c/d Centers]

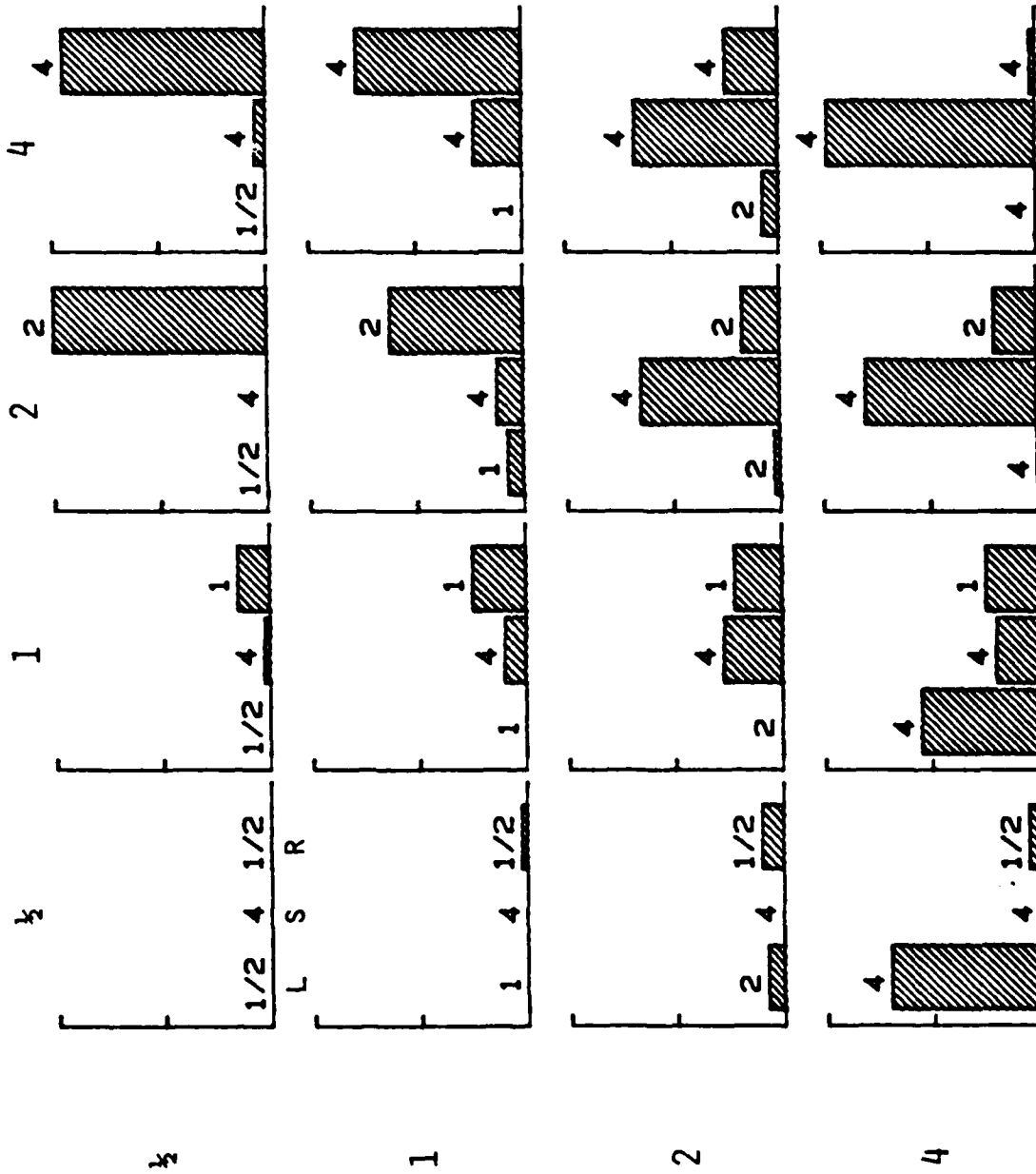


FIGURE 22B (SEE TEXT)

# S.T. [4 c/d Centers]

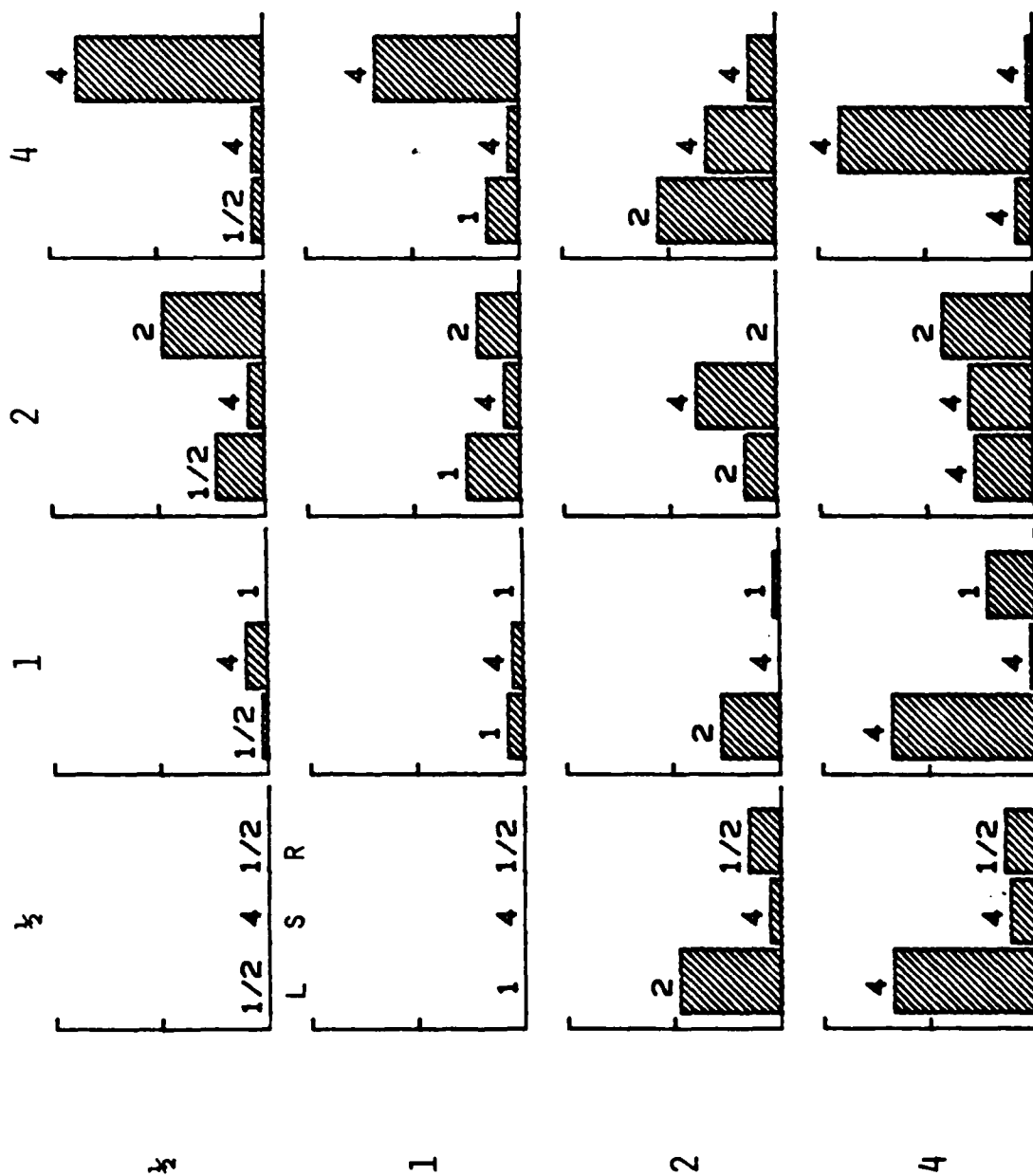


FIGURE 22C (SEE TEXT)

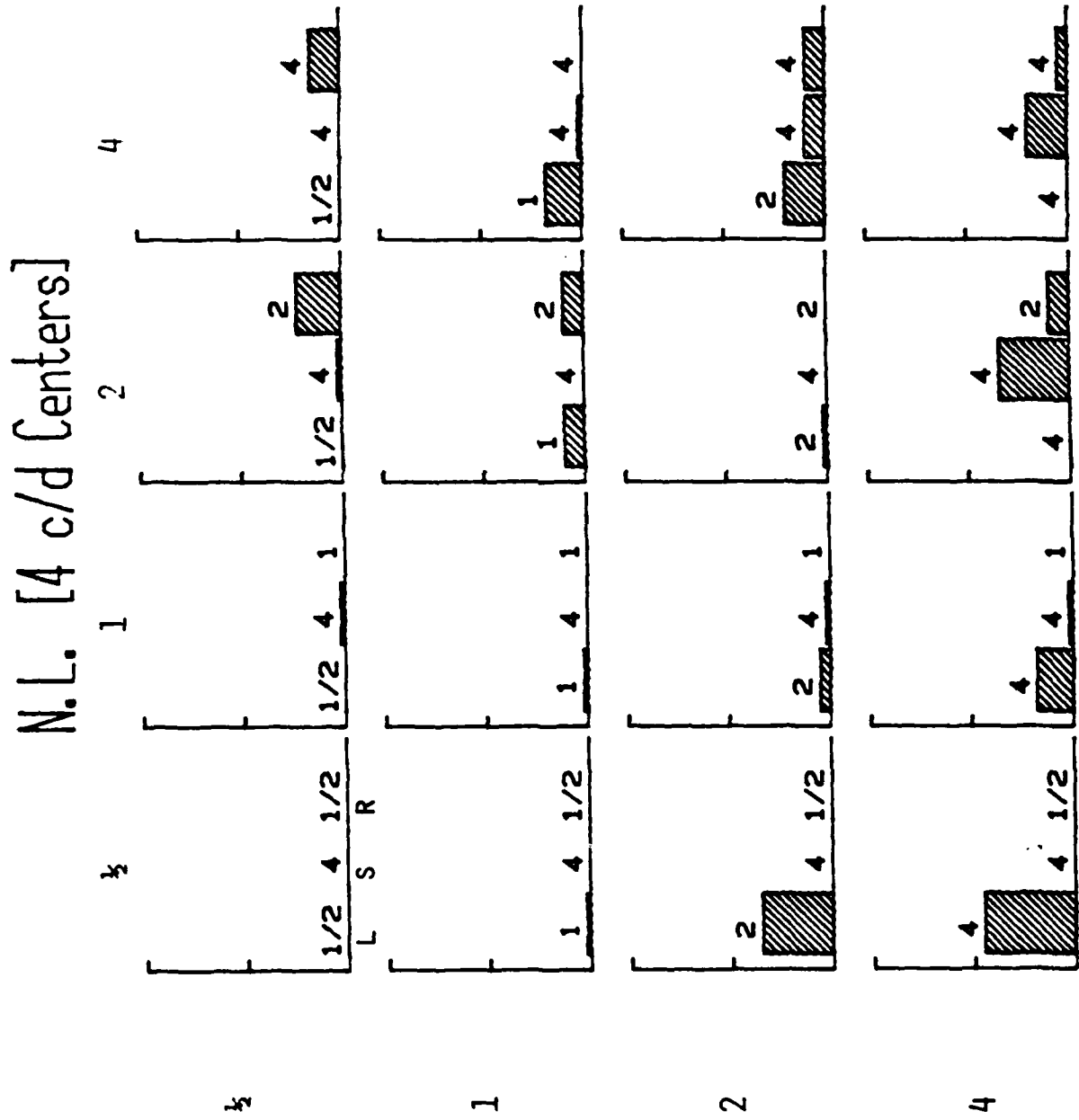


FIGURE 22D (SEE TEXT)

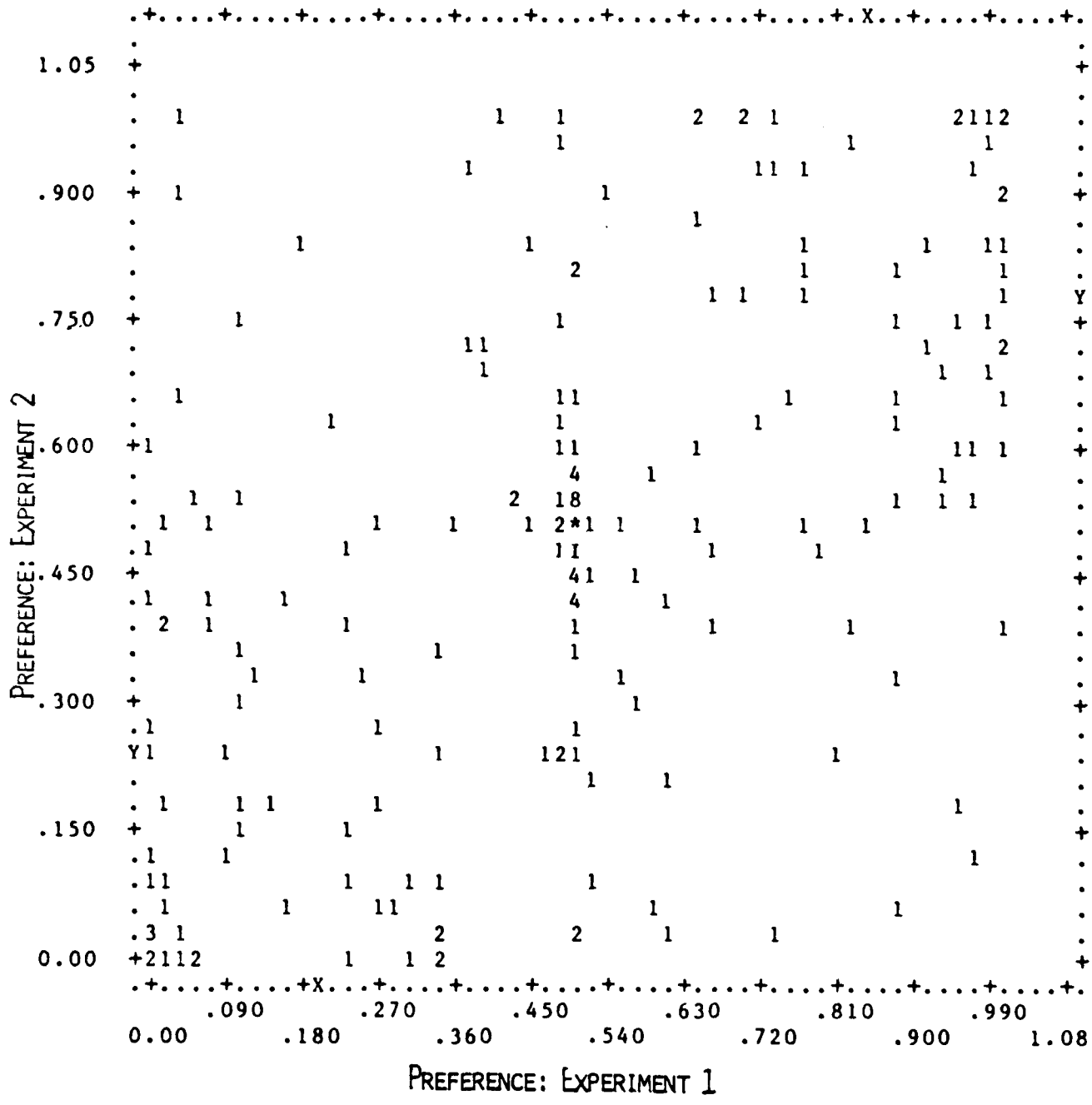


FIGURE 23. SCATTERGRAM PLOTTING CALCULATED PREFERENCES FOR THE FIRST EXPERIMENT (HORIZONTAL AXIS) VS. THE SECOND EXPERIMENT, (VERTICAL AXIS). DIGITS SIGNIFY NUMBER OF DATA POINTS AT A GIVEN LOCUS.



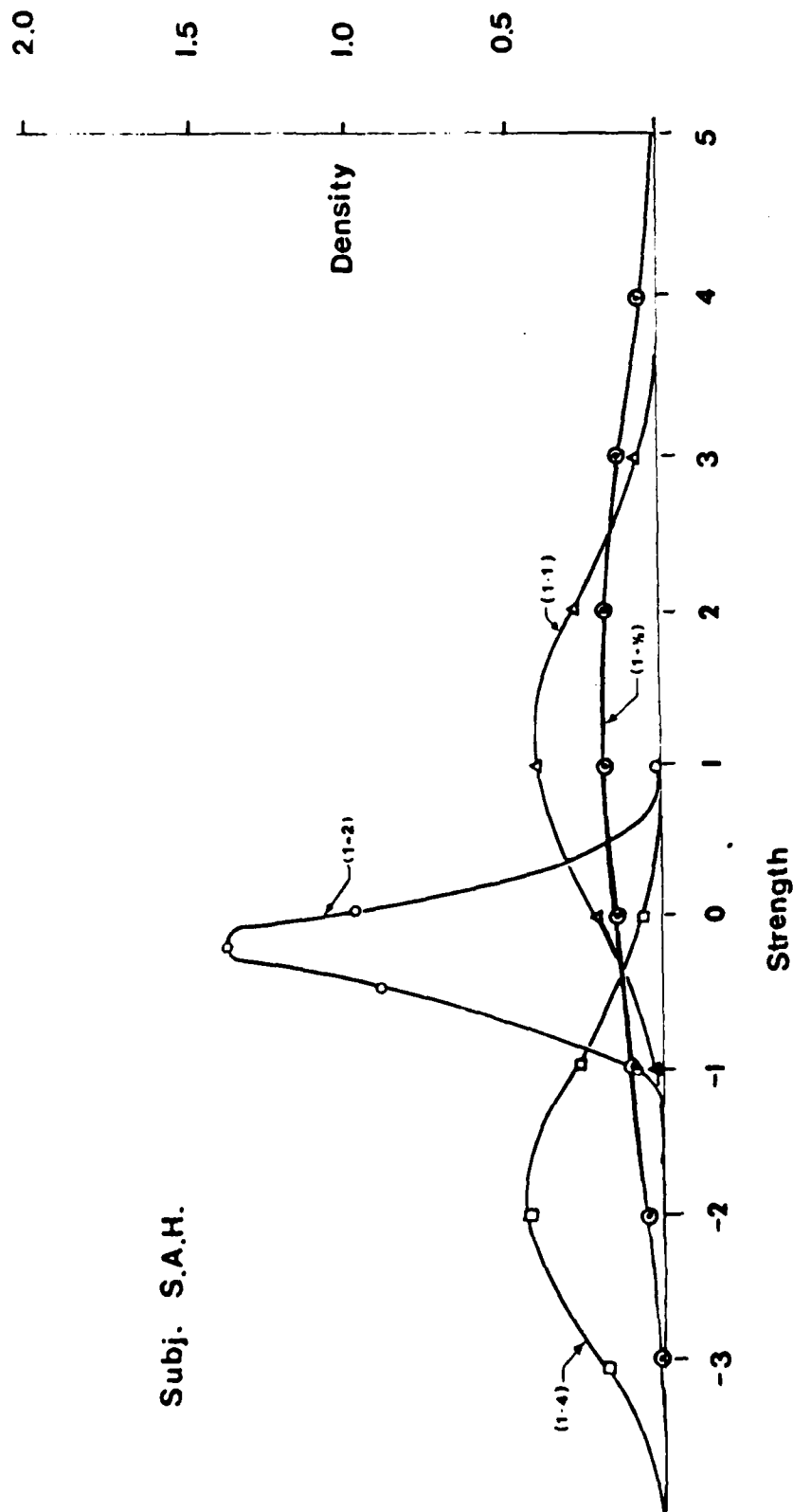


FIGURE 24. ESTIMATED PROBABILITY DENSITY FUNCTIONS FOR THE 4 PAIRS  
WITH 1c/d DOGS ON-FIXATION, SUBJECT S.H.

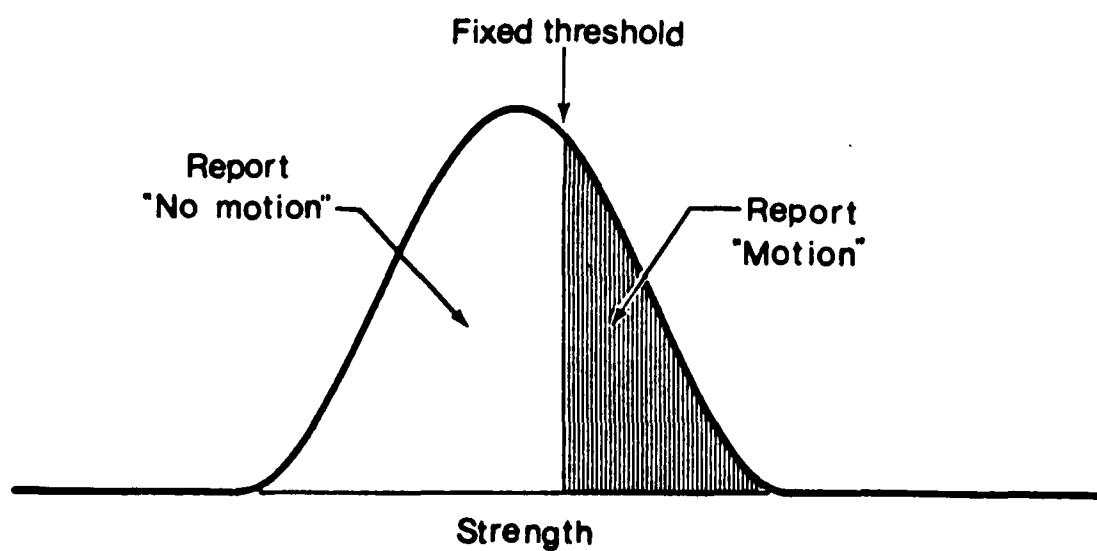


FIGURE 25 FIXED THRESHOLD DECISION RULE AS APPLIED TO PAIR STRENGTH AND MOTION REPORTS. SHADED AREA SHOWS PROBABILITY OF REPORTING MOTION.

Fig. 26 % Seen in motion, Subject S.H.  
Observed versus Predicted

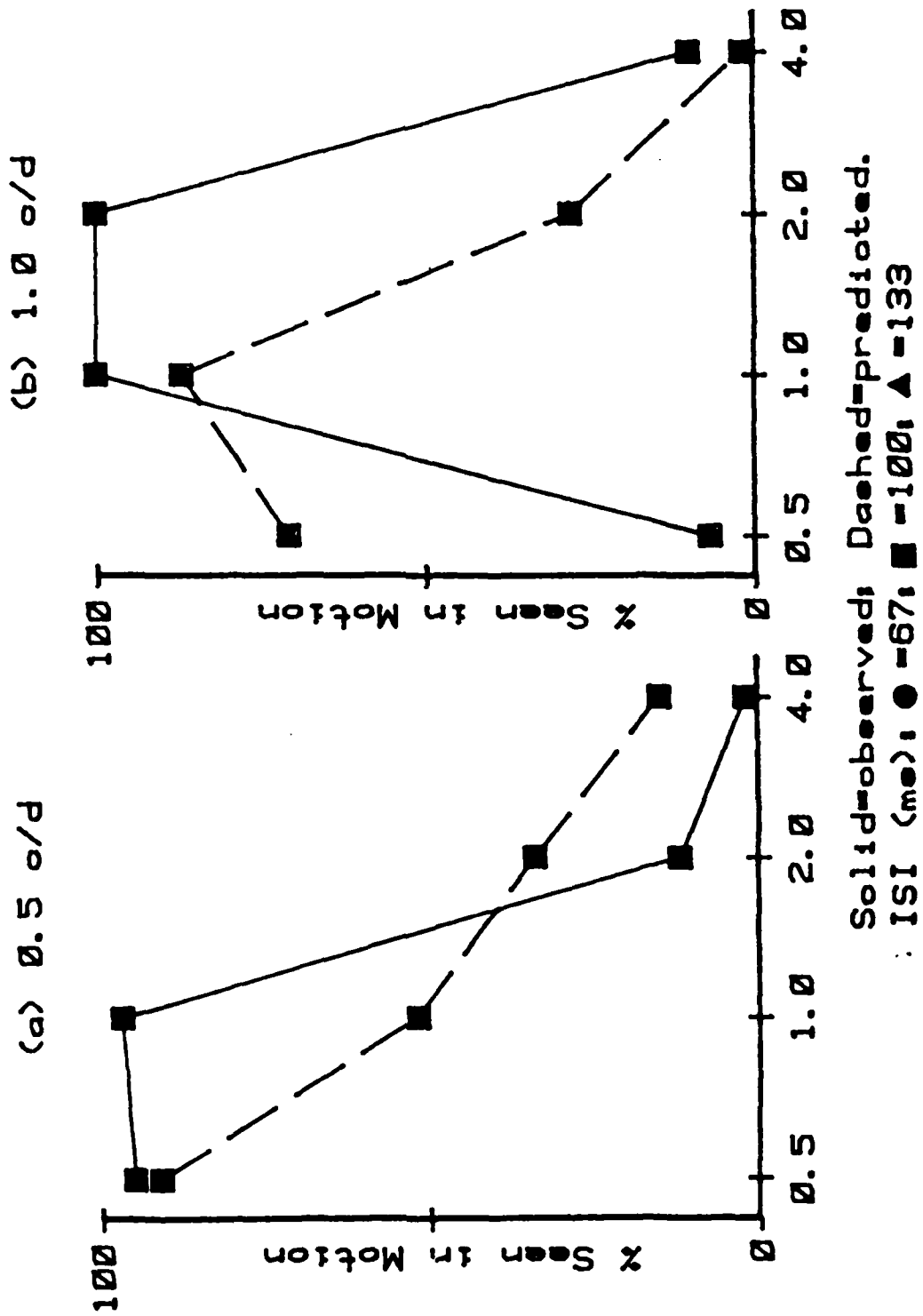


Fig. 26 % Seen in motion, Subject S.H.  
Observed versus Predicted

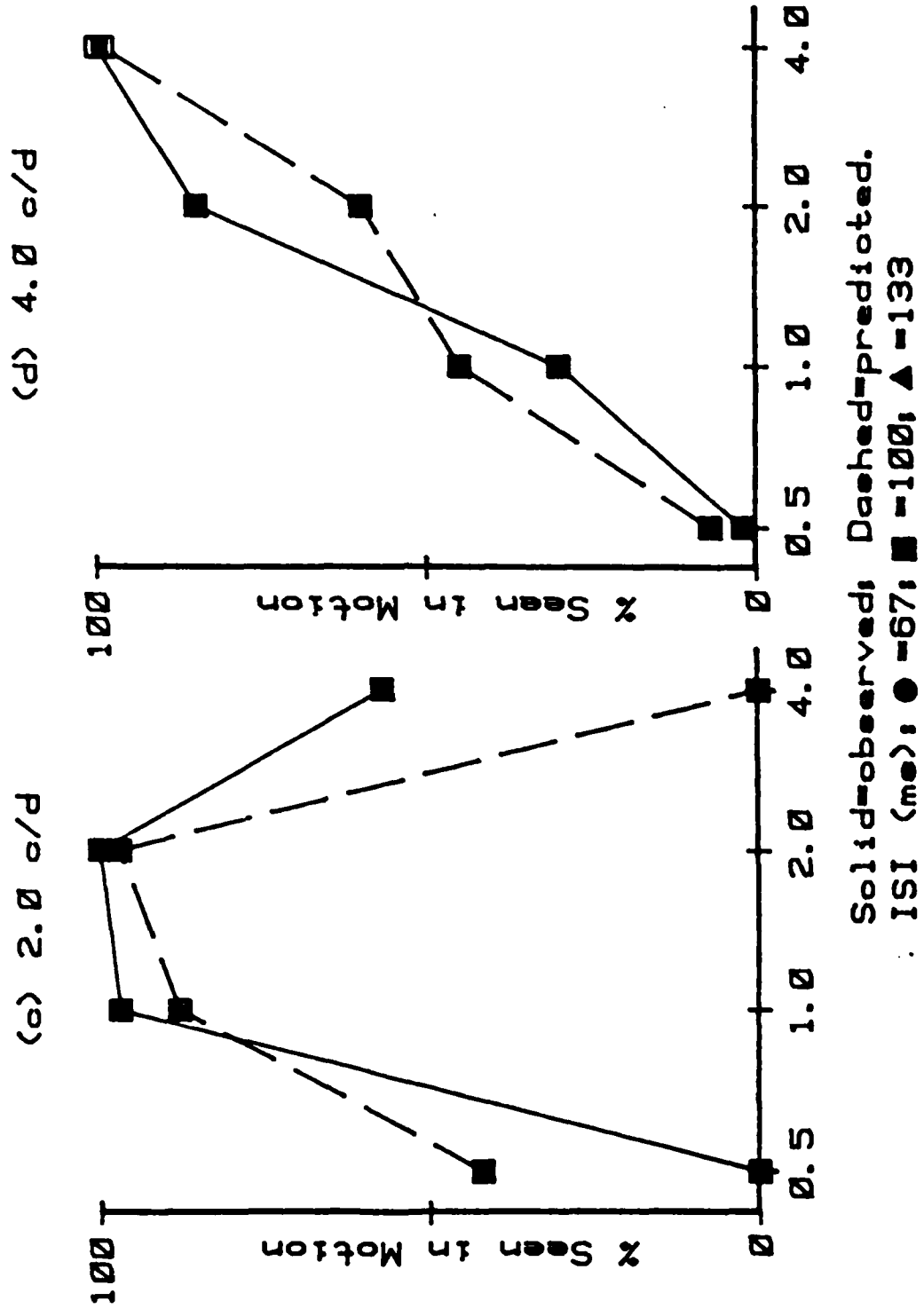


Fig. 27 % Seen in motion, Subject A. S.  
Observed versus Predicted

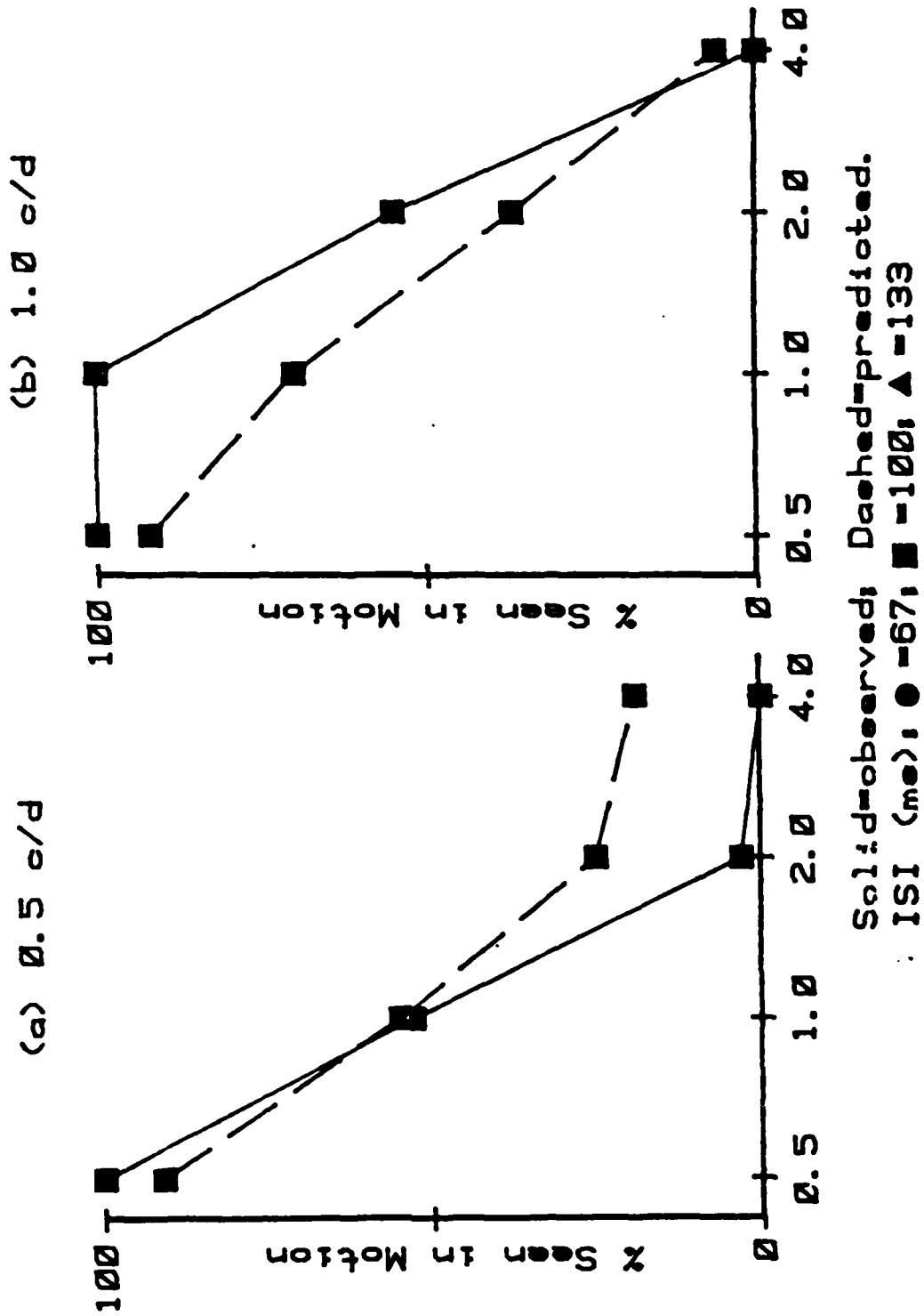
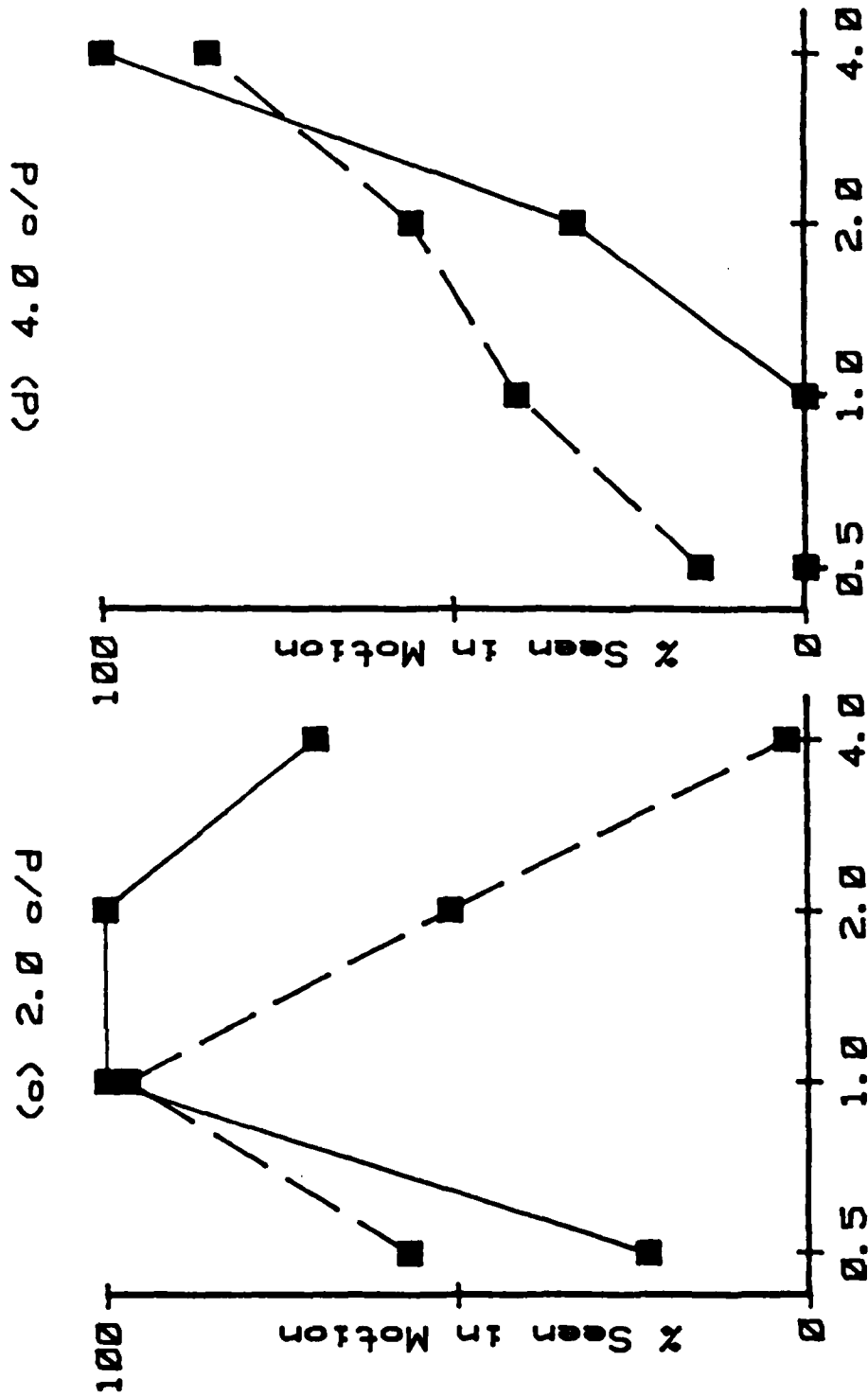


Fig. 27 % Seen in motion, Subject A. S.  
Observed versus Predicted



Solid=observed; Dashed=predicted.  
ISI (m): ● =67; ■ =100; ▲ =133

Fig. 28 % Seen in motion, Subject S.T.  
Observed versus Predicted

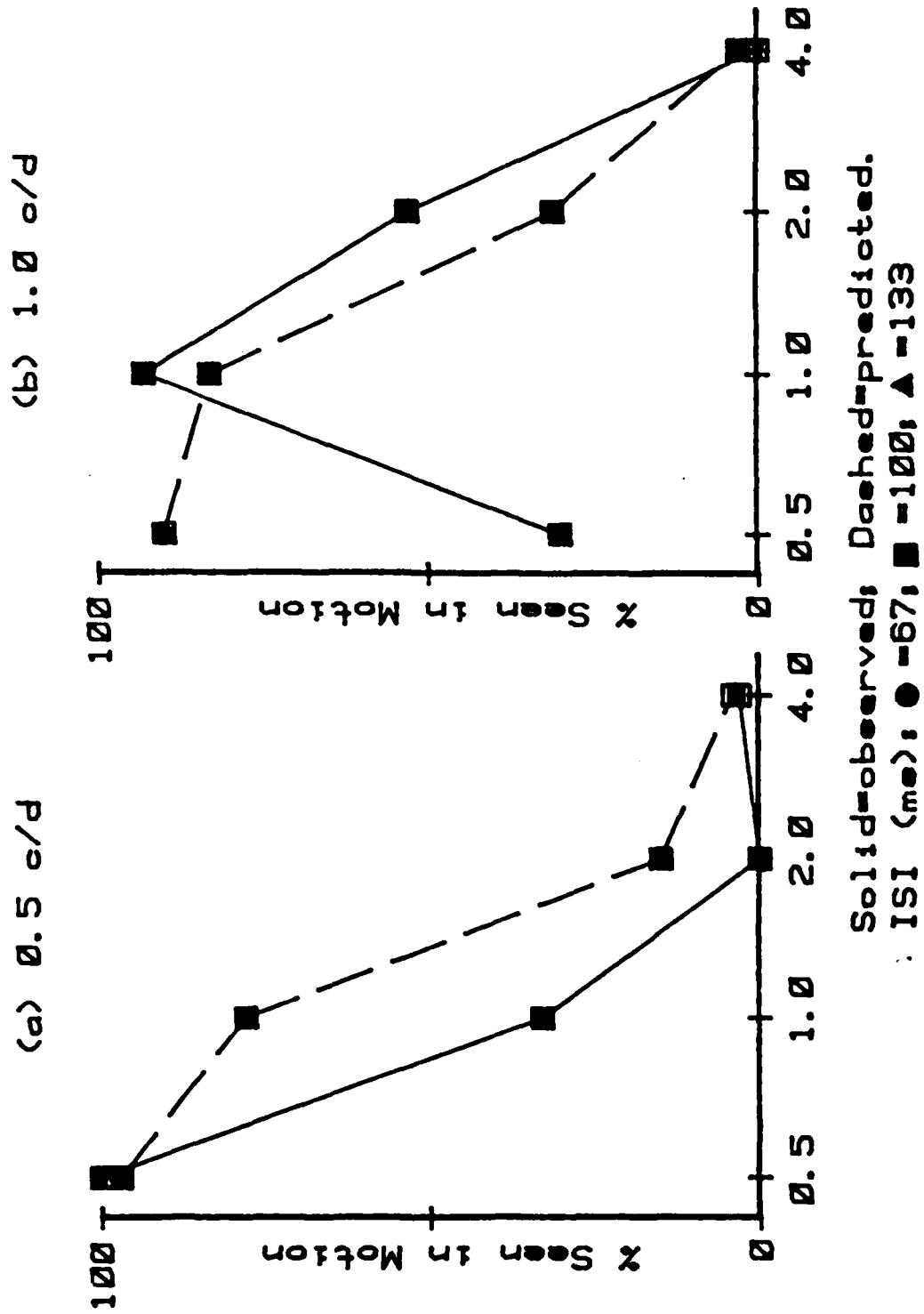


Fig. 28 % Seen in motion, Subject S.T.  
Observed versus Predicted

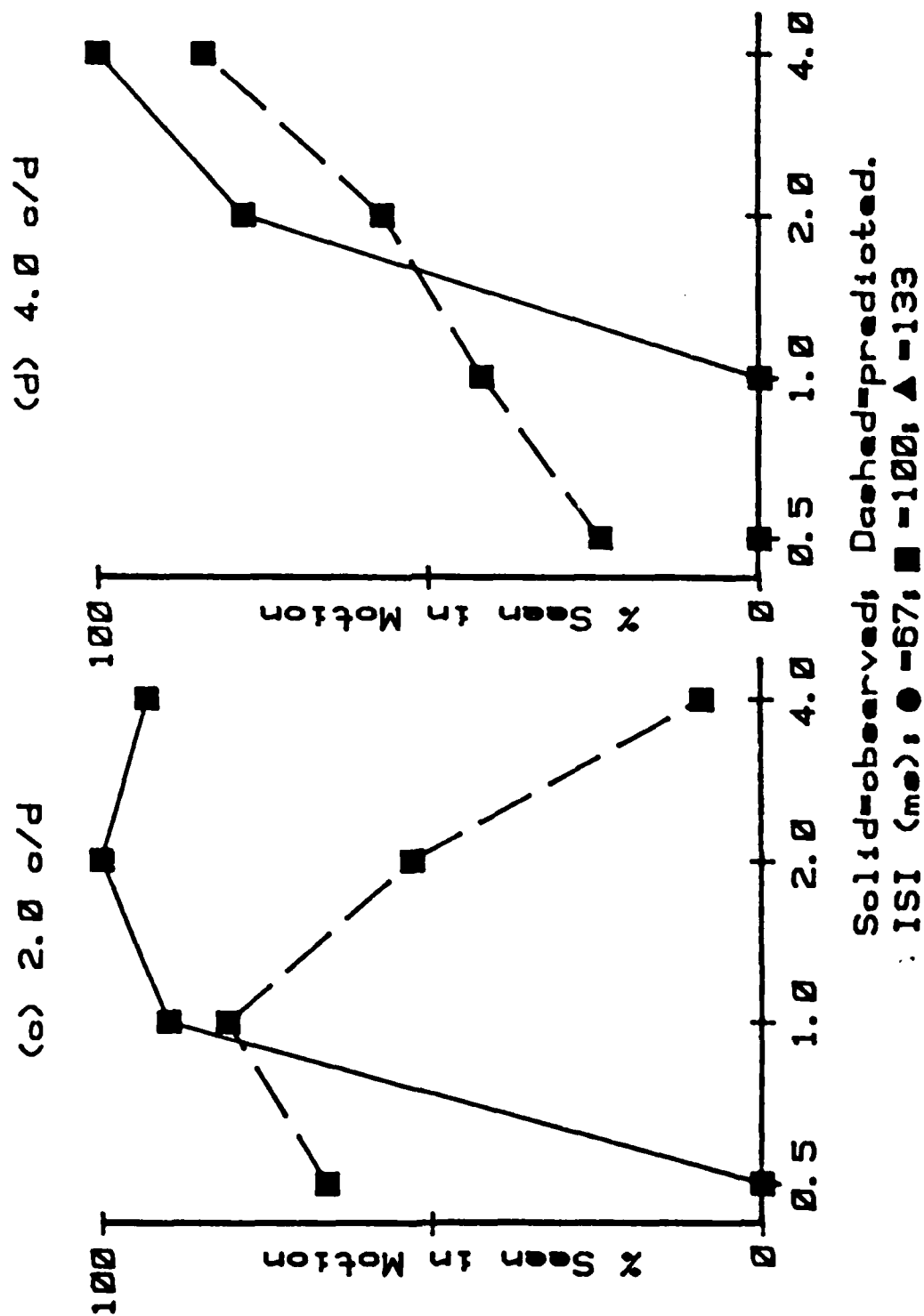




Fig. 29 % Seen in motion, Subject N.L.  
Observed versus Predicted

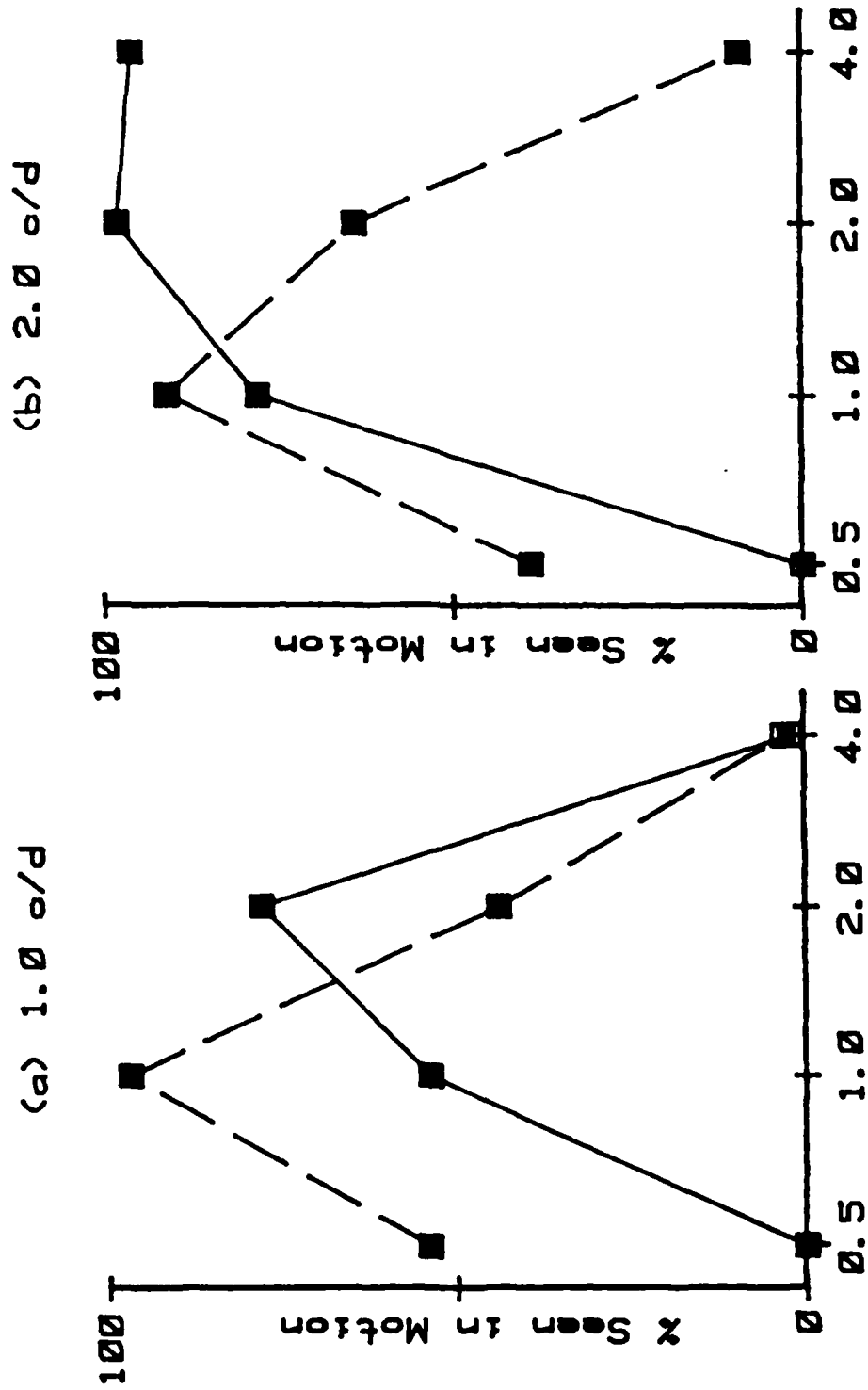


Fig. 30 % Seen in motion, Observed versus Composite, Predicted

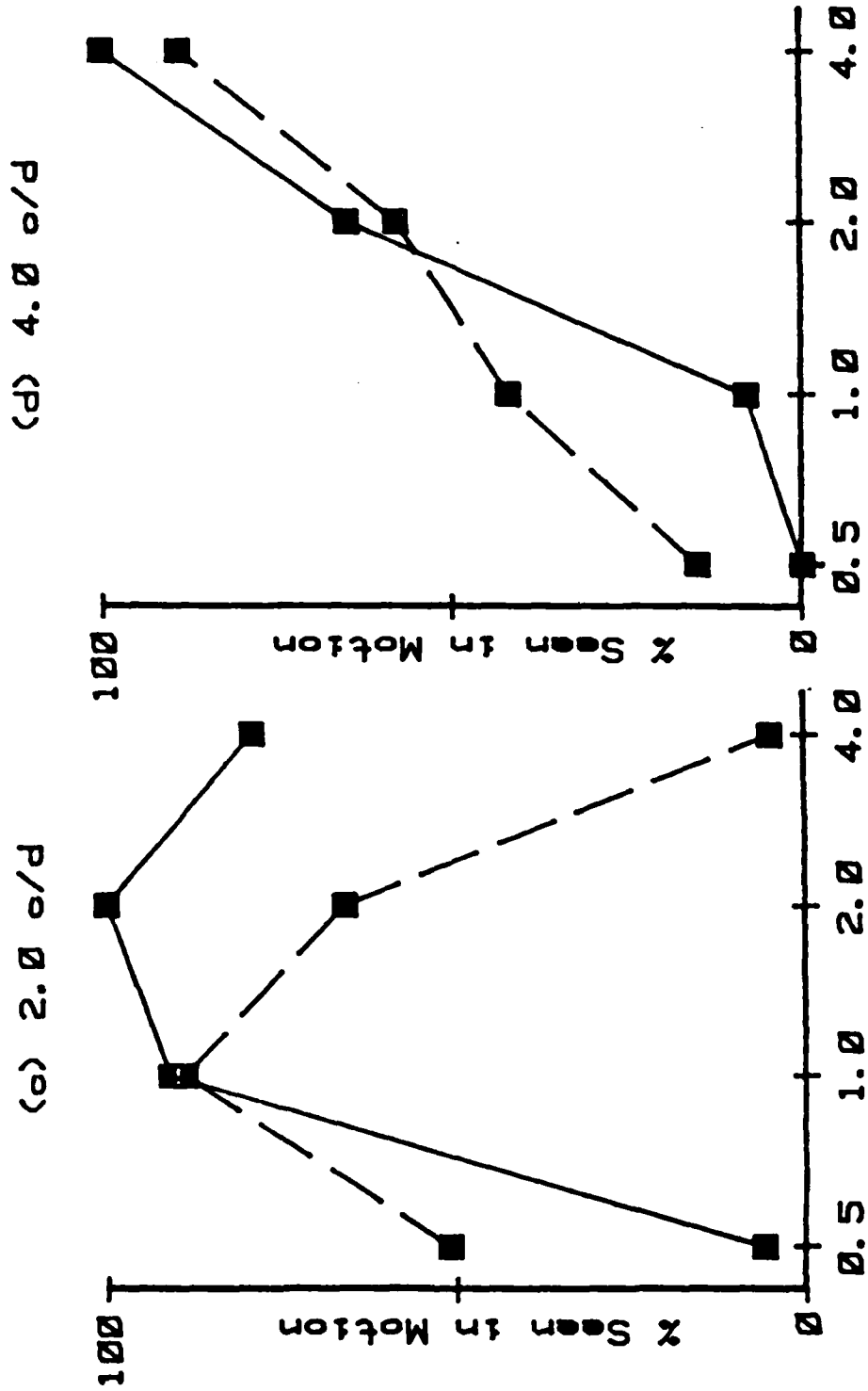
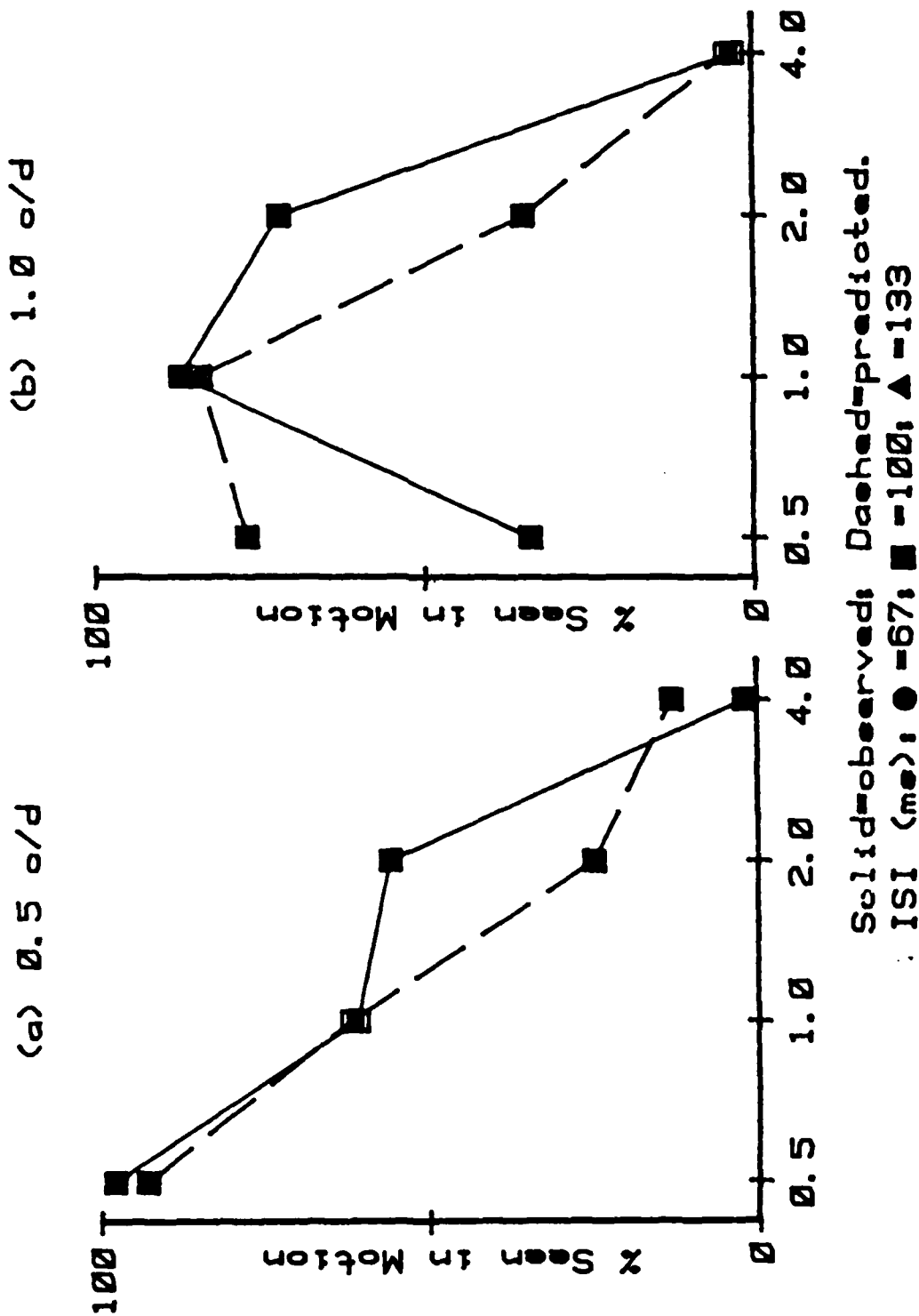


Fig. 30 % Seen in motion, Composite.  
Observed versus Predicted.



# APPENDIX 1 DESCRIPTION OF APPARATUS

The system used to display all patterns described in the preceding experiments is illustrated in the diagram on the following page. The various difference of Gaussians (DOG) waveforms needed are generated by and stored in the memory of a digital computer. Sixty times each second, a crystal controlled clock (also part of the computer) initiates a new frame of the display. The 'new-frame' pulse has three main effects: the sweep generator is triggered to start moving the scope's electron beam horizontally across the screen; the display contrast is selected from a temporal waveform list in computer memory and converted to a voltage level by a 12-bit digital to analog converter; the points of the spatial waveform are converted to voltages by another 12 bit digital to analog converter. All three processes are initiated virtually simultaneously; the selection of display contrast occurs only at the beginning of each frame, and remains constant for the duration of the frame. The conversion of the spatial waveform takes most of the duration of the frame to complete. During this conversion, the sweep generator moves the electron beam across the entire face of the scope, evenly distributing the points of the spatial waveform. The vertical axis of the display scope is modulated by a high-frequency, free running triangle wave generator; the line traced across the screen by the sweep generator is thereby extended to a bar filling the entire screen.

APPENDIX 1

